



# 第02章

# 平面连杆机构

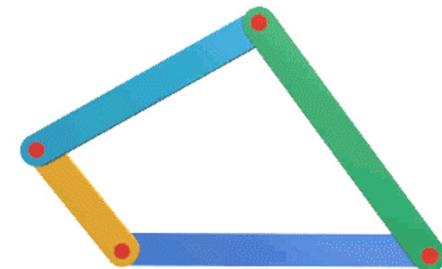
## (上)

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# 本章要点概述

- 平面四杆机构的基本形式、演变及其应用
- 平面四杆机构设计中的共性问题
- 平面四杆机构的设计
- 平面连杆机构的解析综合



# Kinematics

- Degree of Freedom
  - *The number of coordinates needed to define its position in space*
    - One-DOF Mechanism:  $\text{DOF} = 1$
    - Multi-DOF Mechanism:  $\text{DOF} > 1$
    - Structure:  $\text{DOF} = 0$
    - Preloaded Structure:  $\text{DOF} < 1$
- Mechanisms: variants of a linkage
  - *A collection of links and joints, one of which is grounded, and all are interconnected in a way to provide a controlled output in response to one or more inputs.*

## The construction of a linkage

# Links, Joints & Mobility

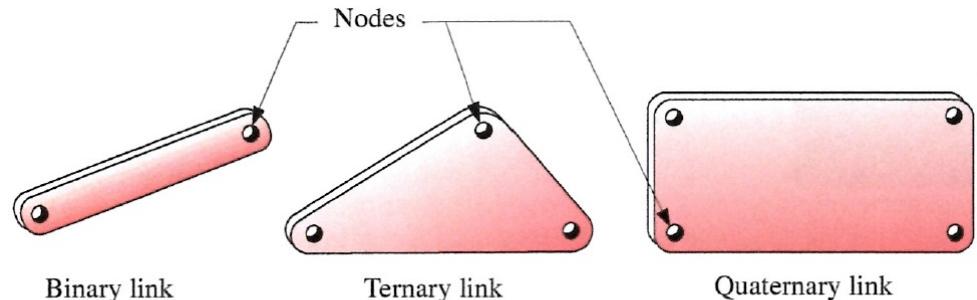
- Link
  - A rigid body of any shape that has some number of attachment points called nodes, that allow multiple links to be connected by joints

- Joints
  - Characterized by their geometry, by the number of DOF they allow between the links they join, and by whether they are held together (closed) by a *force* or by their *form* (geometry)

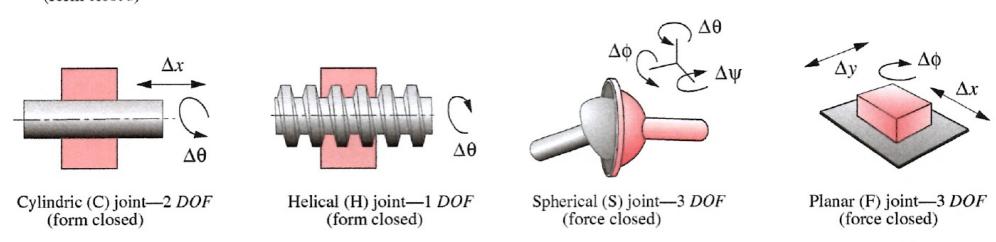
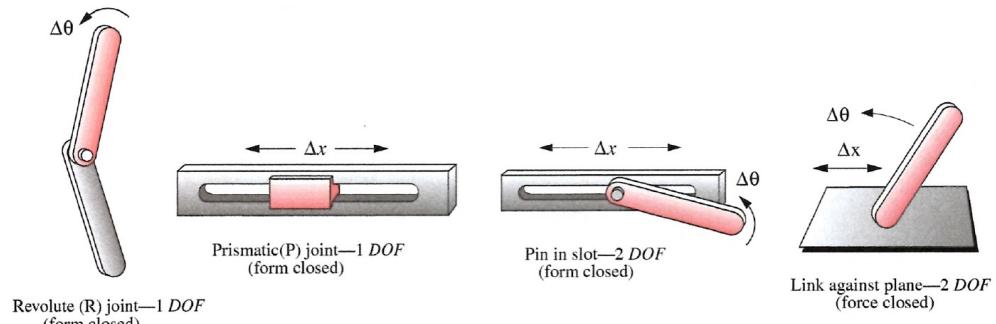
- Kutzbach Equation

$$M = 3(L - 1) - 2J_1 - J_2$$

- M: Mobility (DOF)
- L: the number of links
- J\_1: the number of one-DOF joints
- J\_2: the number of two-DOF joints
- **[Doesn't really work]**



(a) Some links—their names reflect the number of nodes

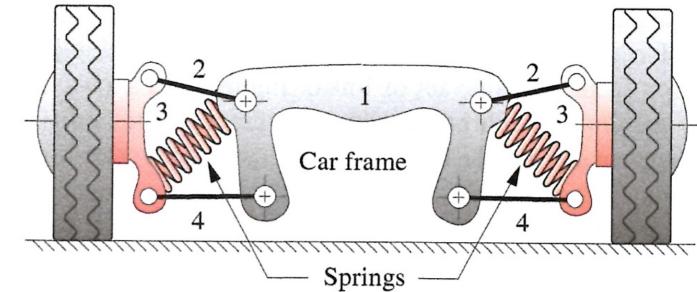


(b) Some joint types—note their DOF and type of closure

## Four binary links connected by four pin joints

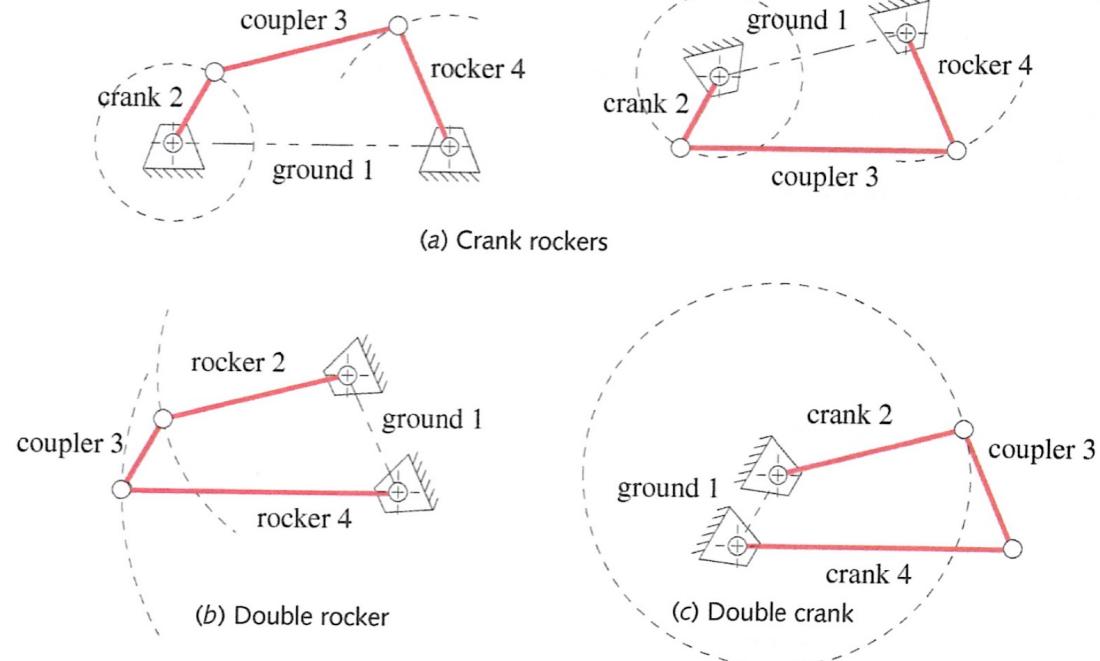
# The Fourbar Linkage

- Grashof Criterion  $S + L \leq P + Q$ 
  - S / L: The length of the shortest / longest link
  - P & Q: The lengths of the other two



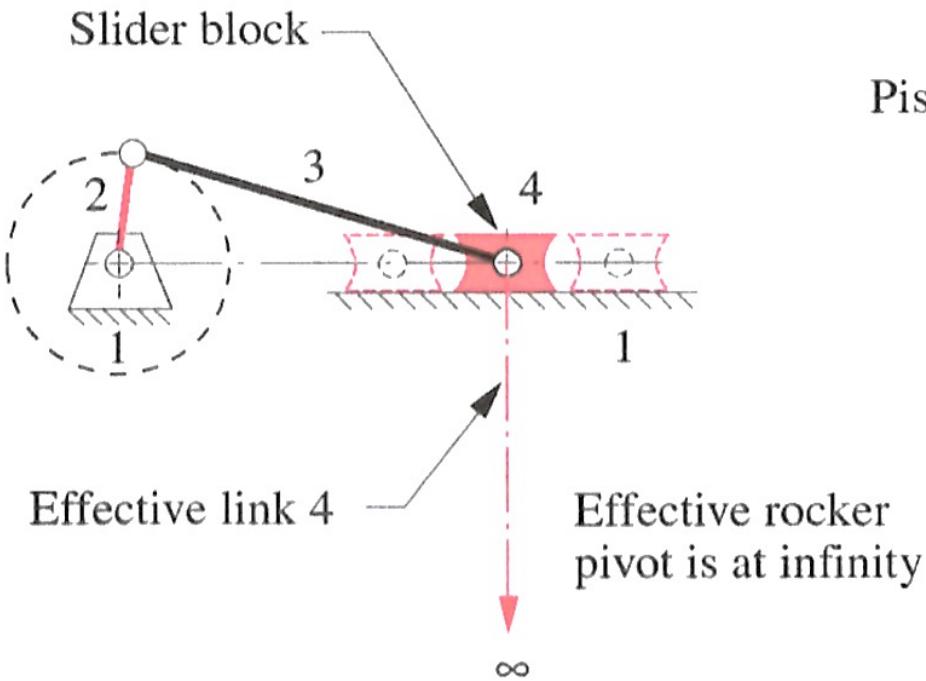
- A Grashof Linkage
  - At least one link can revolve fully
  - It has “change-point” positions when equal [Singularity]

*Fourbar linkage is simply the best linkage of all.*

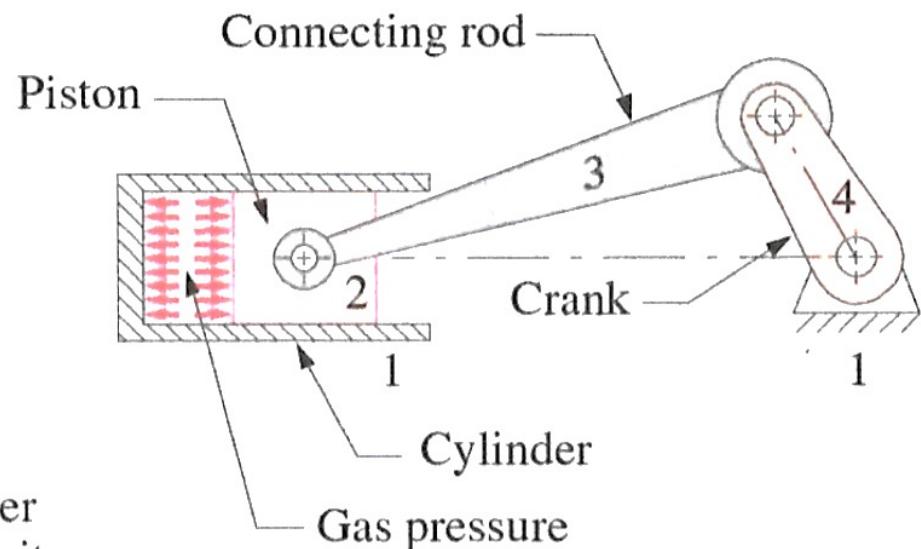


# Fourbar Crank-Slider & Slider-Crank

- If a rocker of a fourbar crank rocker linkage is increased in length indefinitely, it effectively becomes infinite in length and the linkage is transformed into a fourbar crank-slider, which is similar for a fourbar slider-crank.



(a) Crank-slider—crank drives slider

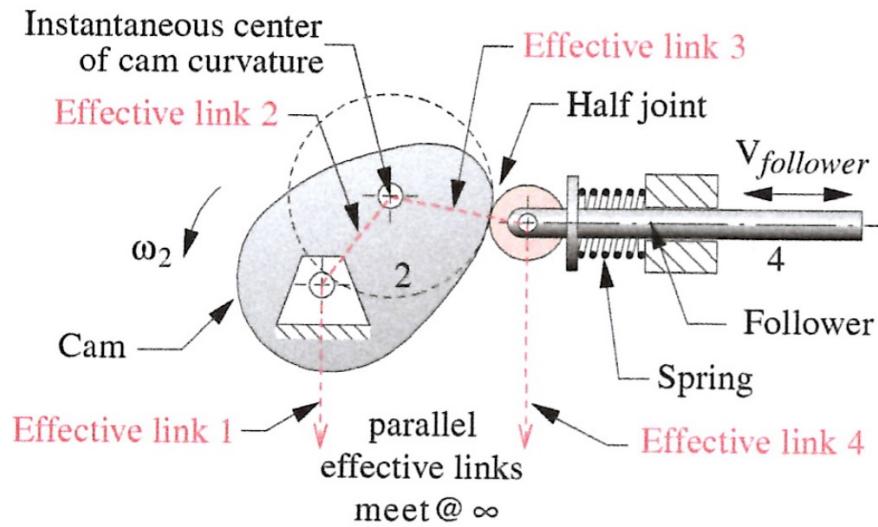


(b) Slider-crank—slider drives crank

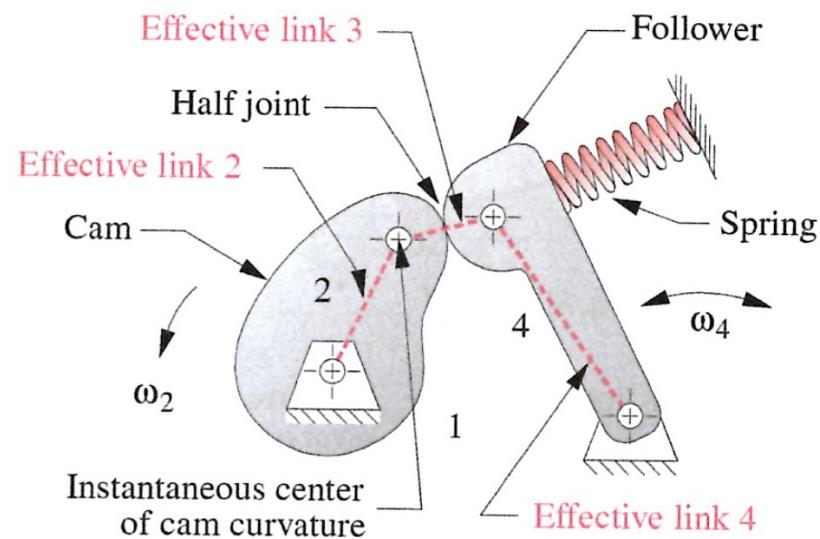
# Another Variant of the Fourbar Linkage

## Cam and Follower

- The crank takes a contoured shape.*
- Can be viewed as a fourbar crank-slider or slider-crank in which the crank and coupler are able to change their lengths as the cam rotates.*



(a) Cam with sliding follower—a variant of a crank-slider

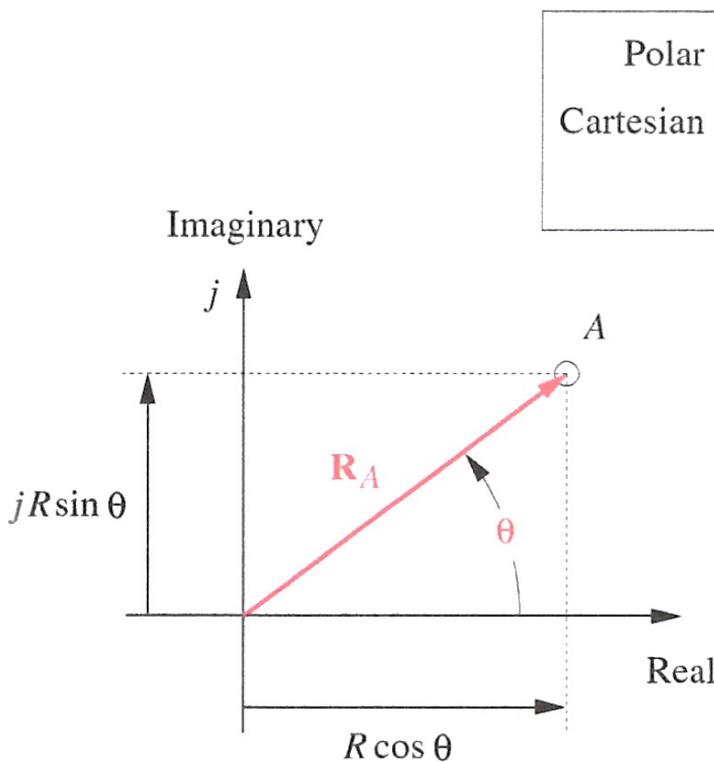


(b) Cam with oscillating follower—a variant of a crank-rocker

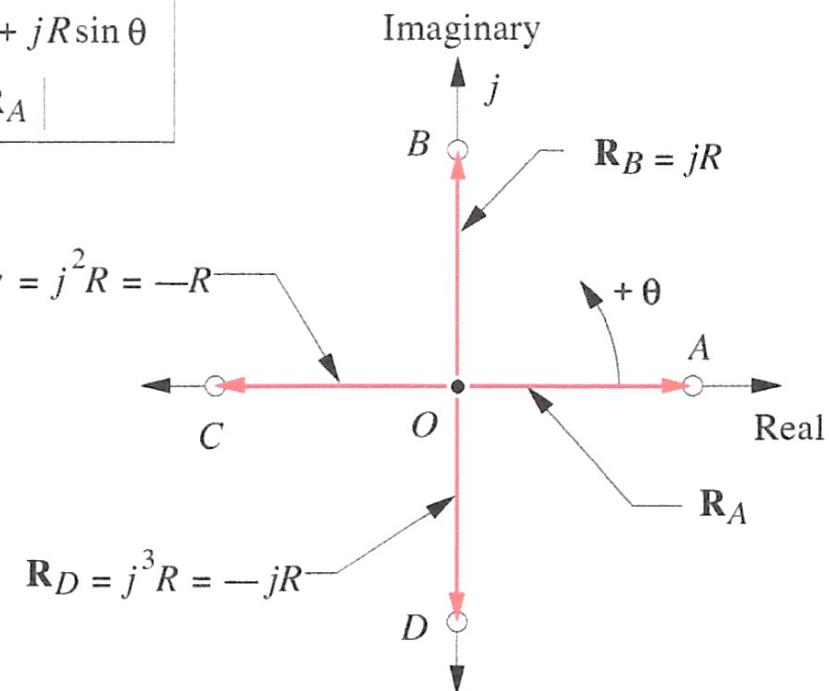
Complex Motion = Translation (x, y) + Rotation (theta)

# Analyzing Linkage Motion

$$e^{\pm j\theta} = \cos \theta \pm j \sin \theta$$



Polar form:	$R e^{j\theta}$
Cartesian form:	$R \cos \theta + j R \sin \theta$
$R =$	$  \mathbf{R}_A  $



(a) Complex number representation of a position vector

(b) Vector rotations in the complex plane

# Analyzing the Fourbar Linkage

Vector Loop  
Equation

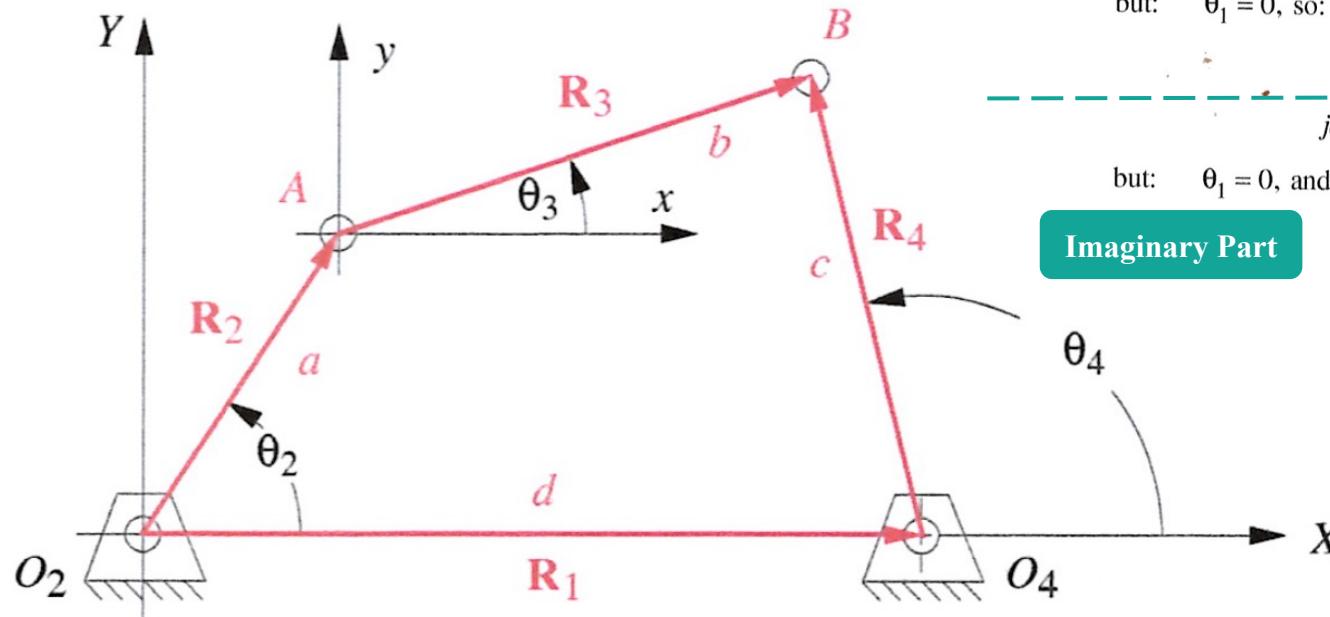
$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0$$



$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} = 0$$



$$a(\cos\theta_2 + j\sin\theta_2) + b(\cos\theta_3 + j\sin\theta_3) - c(\cos\theta_4 + j\sin\theta_4) - d(\cos\theta_1 + j\sin\theta_1) = 0$$



Real Part

but:  $\theta_1 = 0$ , so:

$$a\cos\theta_2 + b\cos\theta_3 - c\cos\theta_4 - d\cos\theta_1 = 0$$

$$a\cos\theta_2 + b\cos\theta_3 - c\cos\theta_4 - d = 0$$

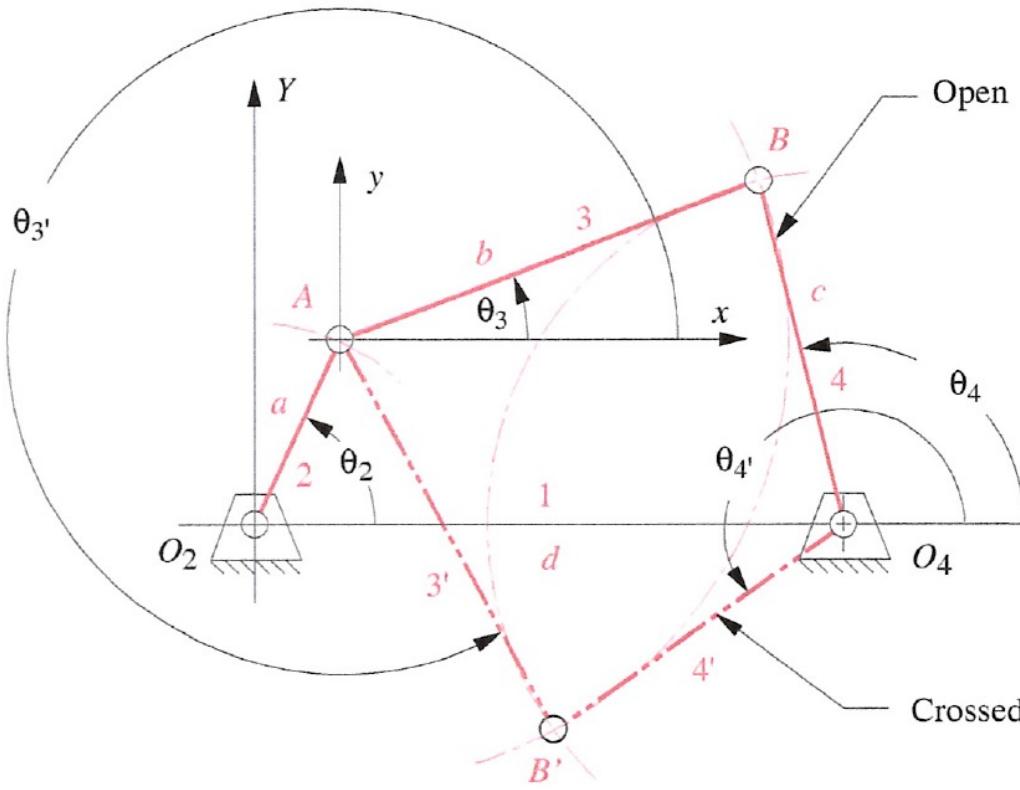
$$jas\theta_2 + jb\sin\theta_3 - jc\sin\theta_4 - jd\sin\theta_1 = 0$$

but:  $\theta_1 = 0$ , and the  $j$ 's divide out, so:

Imaginary Part

$$as\theta_2 + b\sin\theta_3 - c\sin\theta_4 = 0$$

# The Two Solutions to theta\_3/4 of the Fourbar Position Equation



where

$$\theta_{3_{1,2}} = 2 \arctan \left( \frac{-E \pm \sqrt{E^2 - 4DF}}{2D} \right)$$

$$D = \cos \theta_2 - K_1 + K_4 \cos \theta_2 + K_5$$

$$E = -2 \sin \theta_2$$

$$F = K_1 + (K_4 - 1) \cos \theta_2 + K_5$$

and

$$K_1 = \frac{d}{a} \quad K_4 = \frac{d}{b} \quad K_5 = \frac{c^2 - d^2 - a^2 - b^2}{2ab}$$

where:

$$\theta_{4_{1,2}} = 2 \arctan \left( \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \right)$$

$$A = \cos \theta_2 - K_1 - K_2 \cos \theta_2 + K_3$$

$$B = -2 \sin \theta_2$$

$$C = K_1 - (K_2 + 1) \cos \theta_2 + K_3$$

and:

$$K_1 = \frac{d}{a}, \quad K_2 = \frac{d}{c}, \quad K_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac}$$

# Velocity Equation of the Fourbar Linkage

Vector Loop  
Equation

$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0$$



$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} = 0$$



*Differentiate with respect to time to get velocity.*

$$ja e^{j\theta_2} \frac{d\theta_2}{dt} + jb e^{j\theta_3} \frac{d\theta_3}{dt} - jc e^{j\theta_4} \frac{d\theta_4}{dt} = 0$$

but,

$$\frac{d\theta_2}{dt} = \omega_2;$$

$$\frac{d\theta_3}{dt} = \omega_3;$$

$$\frac{d\theta_4}{dt} = \omega_4$$

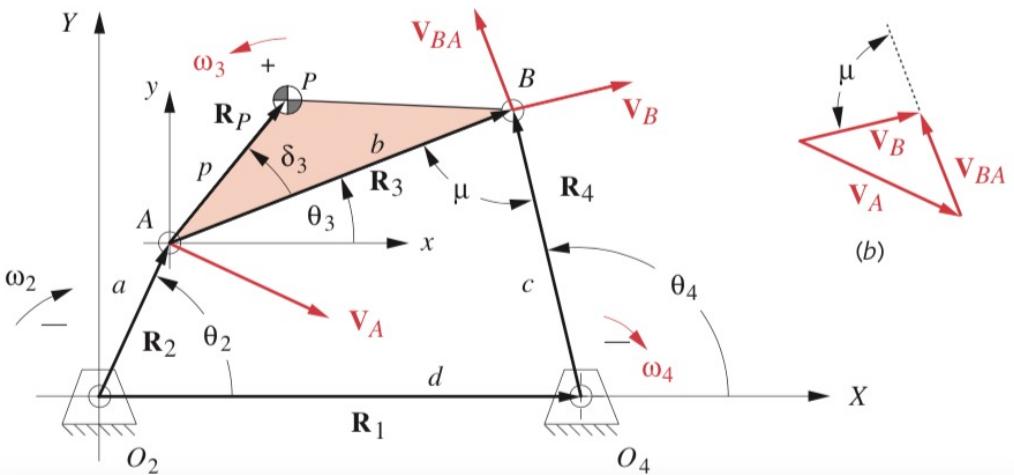
so:

$$ja \omega_2 e^{j\theta_2} + jb \omega_3 e^{j\theta_3} - jc \omega_4 e^{j\theta_4} = 0$$

$$e^{\pm j\theta} = \cos \theta \pm j \sin \theta$$

$$\omega_3 = \frac{a\omega_2}{b} \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)}$$

$$\omega_4 = \frac{a\omega_2}{c} \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)}$$



## Acceleration Equation of the Fourbar Linkage

Vector Loop  
Equation

$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0$$

$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} = 0$$

$$ja\omega_2 e^{j\theta_2} + jb\omega_3 e^{j\theta_3} - jc\omega_4 e^{j\theta_4} = 0$$

*Differentiate with respect to time to get velocity.*

*Differentiate with respect to time to get acceleration.*

$$(j^2 a\omega_2^2 e^{j\theta_2} + ja\alpha_2 e^{j\theta_2}) + (j^2 b\omega_3^2 e^{j\theta_3} + jb\alpha_3 e^{j\theta_3}) - (j^2 c\omega_4^2 e^{j\theta_4} + jc\alpha_4 e^{j\theta_4}) = 0$$

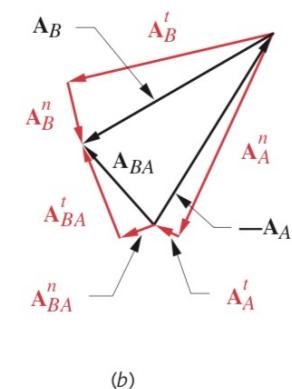
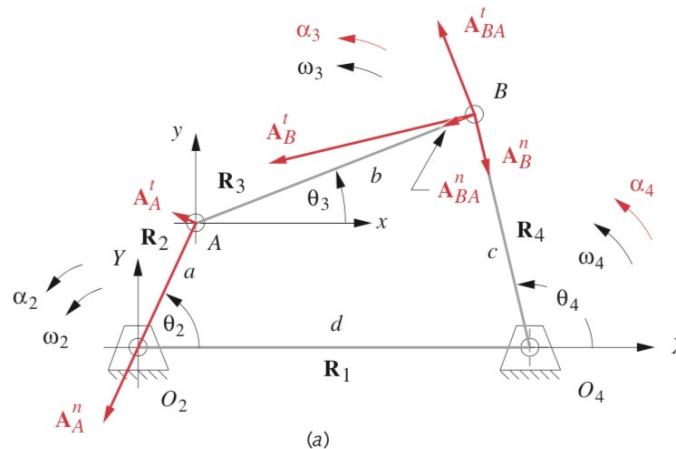
*Rewrite for physical meaning.*

$$\mathbf{A}_A + \mathbf{A}_{BA} - \mathbf{A}_B = 0$$

$$\mathbf{A}_A = (\mathbf{A}_A^t + \mathbf{A}_A^n) = (a\alpha_2 je^{j\theta_2} - a\omega_2^2 e^{j\theta_2})$$

$$\mathbf{A}_{BA} = (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n) = (b\alpha_3 je^{j\theta_3} - b\omega_3^2 e^{j\theta_3})$$

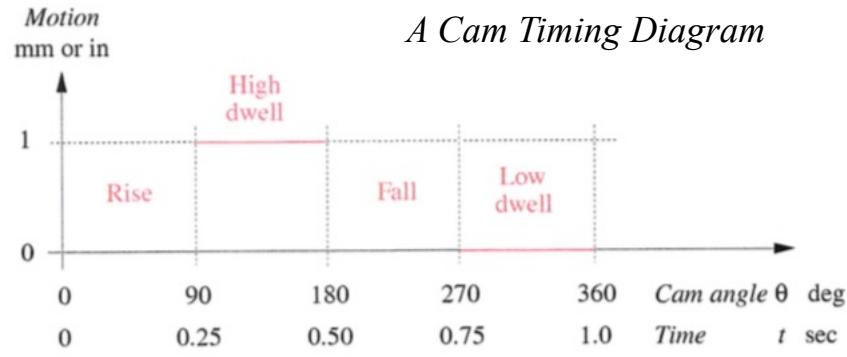
$$\mathbf{A}_B = (\mathbf{A}_B^t + \mathbf{A}_B^n) = (c\alpha_4 je^{j\theta_4} - c\omega_4^2 e^{j\theta_4})$$



# Cam Design & Analysis

- Dwell

- The cessation of follower motion while the cam rotation continues for a portion of the cycle



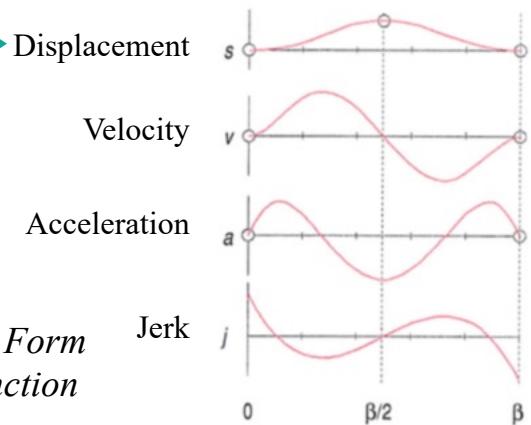
$$s = C_0 + C_1x + C_2x^2 + C_3x^3 + C_4x^4 + C_5x^5 + C_6x^6 + \dots + C_nx^n$$

The crank takes a contoured shape.

General Polynomial Form  
of Displacement Function



svaj Diagram



- Fundamental Law of Cam Design

- The motion function must be piecewise continuous over the entire cam through the second derivative of displacement.*
- No discontinuities in the position, velocity, or acceleration functions over the full cycle of the cam.**

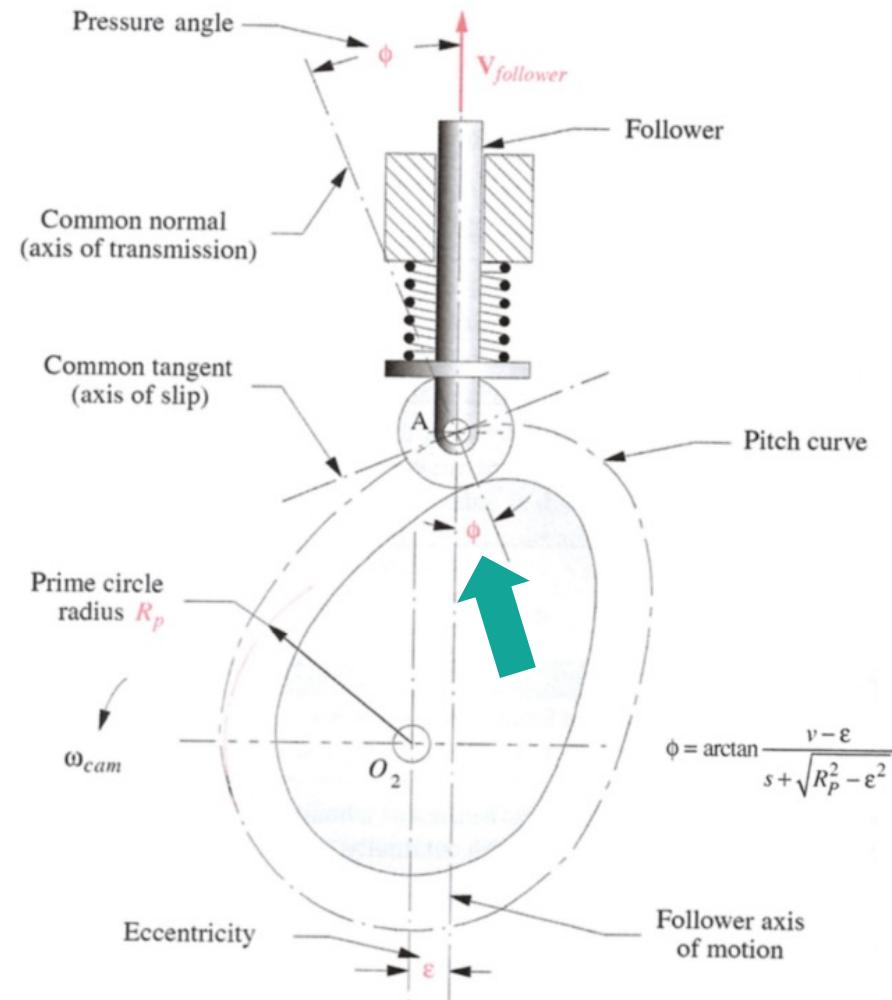
# Pressure Angle

- Definition

- The angle between the velocity of the follower and the common normal at the contact point between cam and follower.

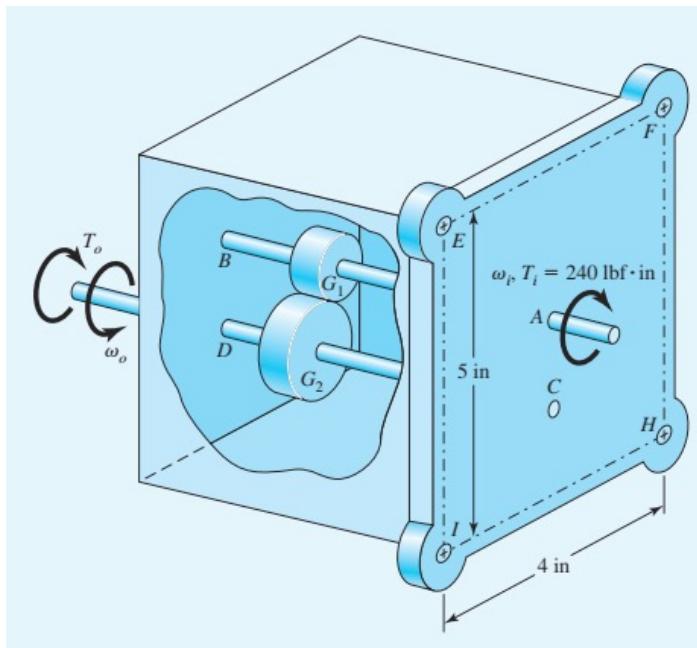
- Rule of thumb

- Keep the maximum pressure angle below about 30 degree for a translating follower.



# Equilibrium & Free-body Diagrams

- **Equilibrium:** A System with Zero Acceleration.
- **Free-body Diagrams:** A means of breaking a complicated problem into manageable segments, analyzing these simple problems, and then, usually, putting the information together again.

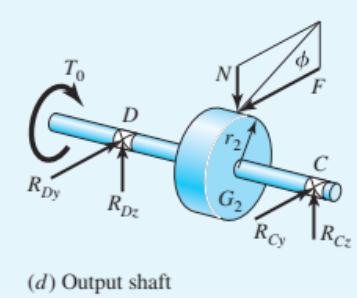
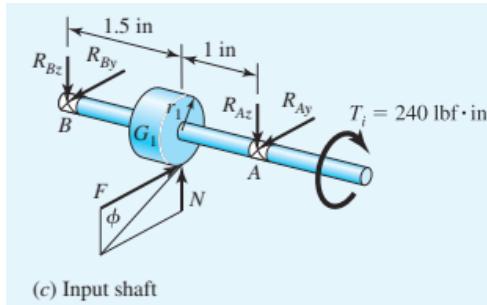


$$\sum \mathbf{F} = 0$$

$$\sum \mathbf{M} = 0$$

$$\begin{aligned}\sum M_x &= F(0.75) - 240 = 0 \\ F &= 320 \text{ lbf}\end{aligned}$$

$$\begin{aligned}R_{Ay} &= 192 \text{ lbf}, R_{Az} = 69.9 \text{ lbf}, \\ R_{By} &= 128 \text{ lbf}, R_{Bz} = 46.6 \text{ lbf}, \\ R_{Cy} &= 192 \text{ lbf}, R_{Cz} = 69.9 \text{ lbf}, \\ R_{Dy} &= 128 \text{ lbf}, R_{Dz} = 46.6 \text{ lbf}, \\ T_o &= 480 \text{ lbf} \cdot \text{in.}\end{aligned}$$



# Load Analysis

- **Newton's First Law (Governing Assumption)**

- *A body at rest tends to remain at rest and a body in motion at constant velocity will tend to maintain that velocity unless acted upon by an external force.*

- **Newton's Second Law (Calculated Guess)**

- *The time rate of change of momentum of a body is equal to the magnitude of the applied force and acts in the direction of the force.*

- **Newton's Third Law (Interactive Analysis)**

- *When two particles interact, a pair of equal and opposite reaction forces will exist at their contact point. This force pair will have the same magnitude and act along the same direction line, but have opposite sense.*

$$\sum \mathbf{F} = m\mathbf{a}$$

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

$$\sum F_z = ma_z$$

$$\sum \mathbf{M}_G = \dot{\mathbf{H}}_G$$

$$\mathbf{H}_G = I_x \omega_x \hat{\mathbf{i}} + I_y \omega_y \hat{\mathbf{j}} + I_z \omega_z \hat{\mathbf{k}}$$

$I_x$ ,  $I_y$ , and  $I_z$  are the principal centroidal mass moments of inertia

These 6 equations can be written for each rigid body in a 3-D system

$$\begin{aligned}\sum M_x &= I_x \alpha_x - (I_y - I_z) \omega_y \omega_z \\ \sum M_y &= I_y \alpha_y - (I_z - I_x) \omega_z \omega_x \\ \sum M_z &= I_z \alpha_z - (I_x - I_y) \omega_x \omega_y\end{aligned}$$

Moments      Angular Accelerations

Needs to be solved simultaneously for the forces and moments

Total # of Load Eqs. to solve for a system



# of rigid bodies the system



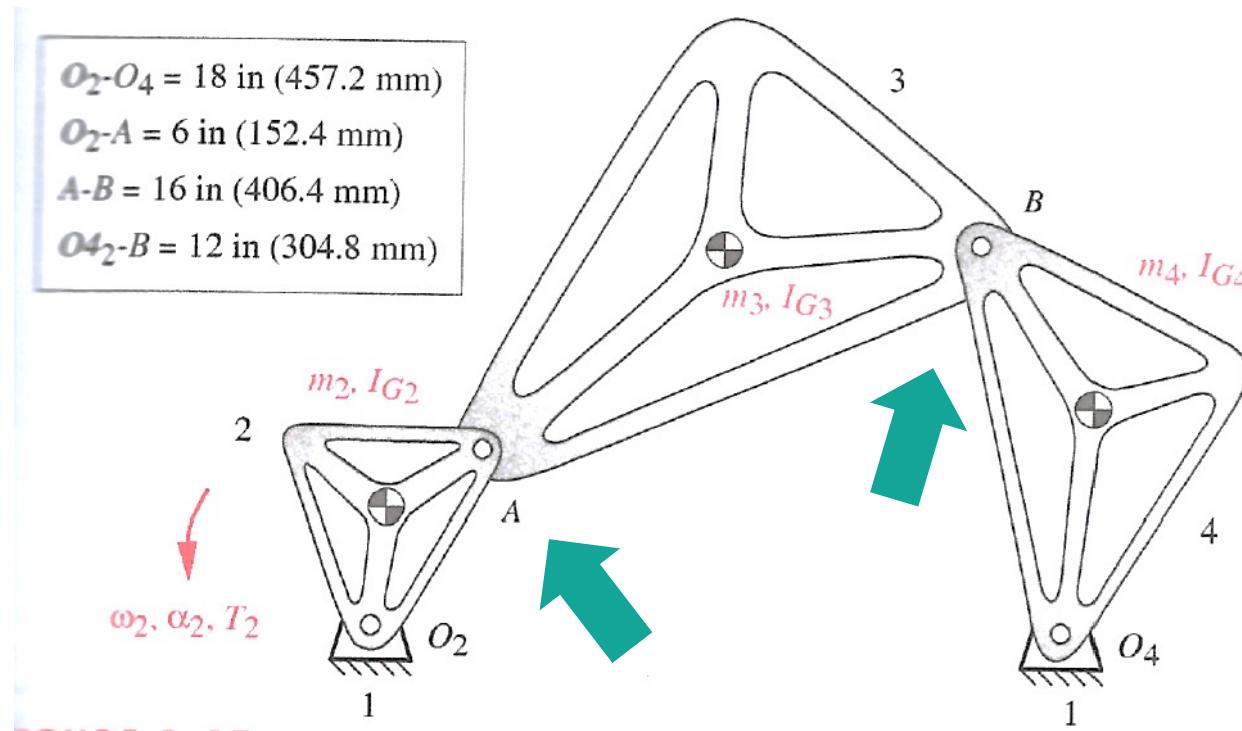
(3 Force Eqs. + 3 Moment Eqs.)



Reaction Eqs.

# Fourbar Linkage Loading Analysis

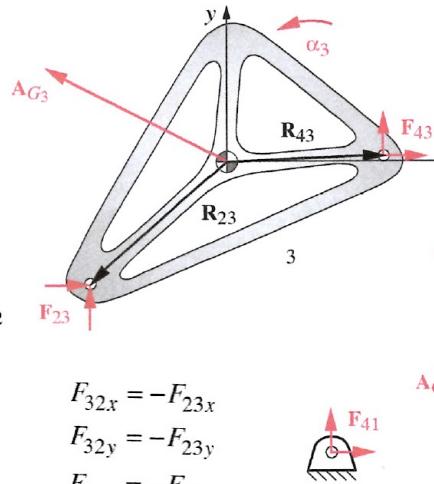
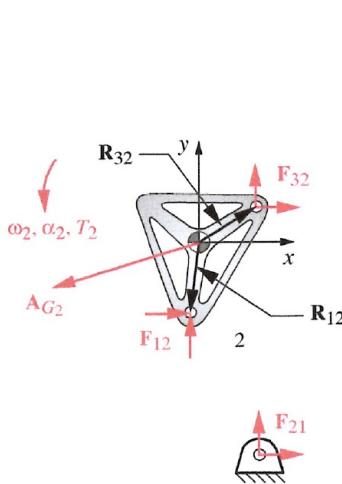
- [Problem] Determine the theoretical rigid body **forces** acting in two dimensions on the fourbar linkage.
- [Given] The linkage geometry, masses, and mass moments of inertia are known and the linkage is driven at up to 120 rpm by a speed-controlled electric motor.



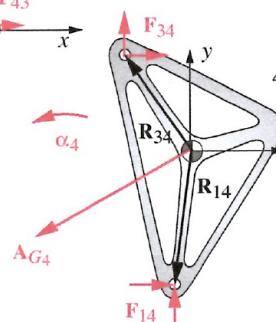
# Free-Body Diagrams

Given and Assumed Data

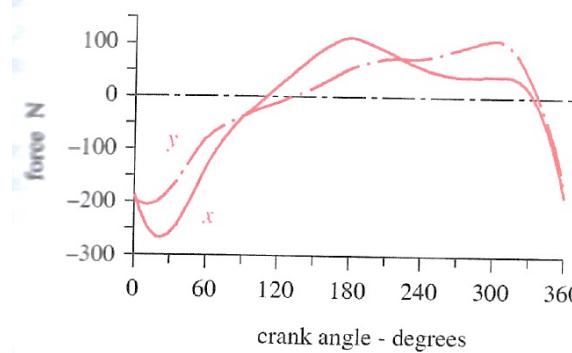
Variable	Value	Unit
$\theta_2$	30.00	deg
$\omega_2$	120.00	rpm
$mass_2$	0.525	kg
$mass_3$	1.050	kg
$mass_4$	1.050	kg
$I_{cg2}$	0.057	$\text{kg}\cdot\text{m}^2$
$I_{cg3}$	0.011	$\text{kg}\cdot\text{m}^2$
$I_{cg4}$	0.455	$\text{kg}\cdot\text{m}^2$
$R_{12x}$	-46.9	mm
$R_{12y}$	-71.3	mm
$R_{32x}$	85.1	mm
$R_{32y}$	4.9	mm
$R_{23x}$	-150.7	mm
$R_{23y}$	-177.6	mm
$R_{43x}$	185.5	mm
$R_{43y}$	50.8	mm
$R_{14x}$	-21.5	mm
$R_{14y}$	-100.6	mm
$R_{34x}$	-10.6	mm
$R_{34y}$	204.0	mm



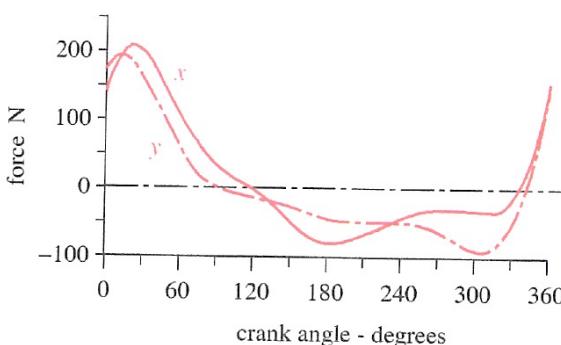
$$\begin{aligned} F_{32x} &= -F_{23x} \\ F_{32y} &= -F_{23y} \\ F_{34x} &= -F_{43x} \\ F_{34y} &= -F_{43y} \end{aligned}$$



x and y forces on joint 12



x and y forces on joint 14



For link 2:

$$\sum F_x = F_{12x} + F_{32x} = m_2 a_{G2x}$$

$$\sum F_y = F_{12y} + F_{32y} = m_2 a_{G2y}$$

$$\sum M_z = T_2 + (R_{12x}F_{12y} - R_{12y}F_{12x}) + (R_{32x}F_{32y} - R_{32y}F_{32x}) = I_{G2}\alpha_2$$

For link 3:

$$\sum F_x = F_{23x} + F_{43x} = m_3 a_{G3x}$$

$$\sum F_y = F_{23y} + F_{43y} = m_3 a_{G3y}$$

$$\sum M_z = (R_{23x}F_{23y} - R_{23y}F_{23x}) + (R_{43x}F_{43y} - R_{43y}F_{43x}) = I_{G3}\alpha_3$$

For link 4:

$$\sum F_x = F_{14x} + F_{34x} = m_4 a_{G4x}$$

$$\sum F_y = F_{14y} + F_{34y} = m_4 a_{G4y}$$

$$\sum M_z = (R_{14x}F_{14y} - R_{14y}F_{14x}) + (R_{34x}F_{34y} - R_{34y}F_{34x}) = I_{G4}\alpha_4$$

# Vibration Loading

- All real elements of any material have elasticity
  - Thus act as spring when subjected to forces
    - Causing deflection to produce additional forces to be generated from the inertial forces associated with the vibratory movement of elements;
    - If clearances allow contact of mating parts, may generate impact (shock) loads during their vibrations.
- How to predict?
  - Modern finite element (FEA) or boundary element (BEA) analysis techniques are good ways to model and calculate
    - Break up the assembly into a large number of discrete elements
    - Limited by time and the computing resources available
  - Field or Scaled experimentations
- How to eliminate?
  - Better Design or Better Design Engineers

# Natural Frequency & Dynamic Forces

- A system's lowest frequency (*a calculated estimation*)
  - Usually creates the largest magnitude of vibration

undamped fundamental natural frequency  $\omega_n$ :  $\omega_n = \sqrt{\frac{k}{m}}$

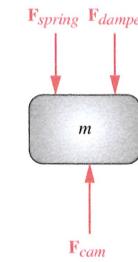
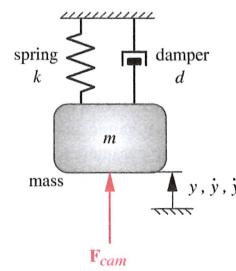
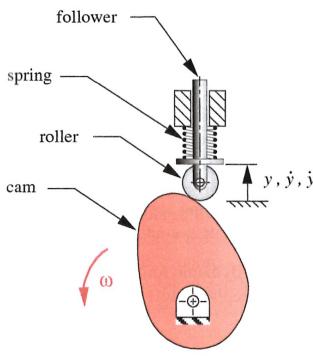
$$f_n = \frac{1}{2\pi} \omega_n$$

$$\begin{aligned}\sum F_y &= ma = m\ddot{y} \\ F_{cam} - F_{spring} - F_{damper} &= m\ddot{y} \\ F_{cam} &= m\ddot{y} + d\dot{y} + ky\end{aligned}$$

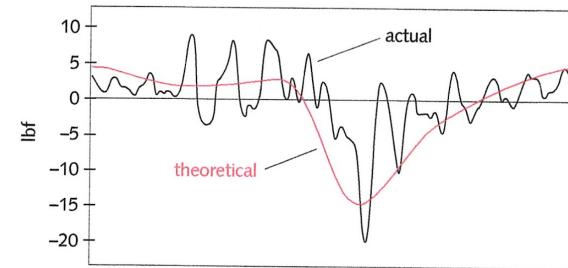
## Metrics

- Spring Constant  $k$
- Damping  $d$      $d = \frac{F}{\dot{y}}$

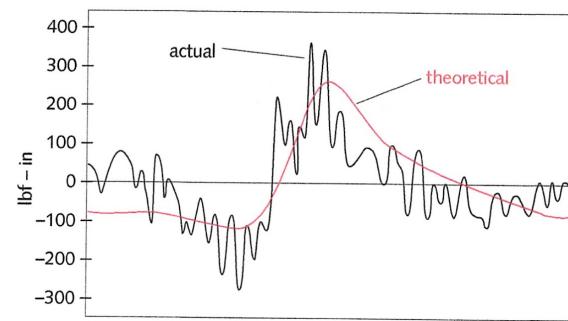
$$k = \frac{F}{y}$$



(a) Theoretical and actual dynamic force in x direction at crank pivot



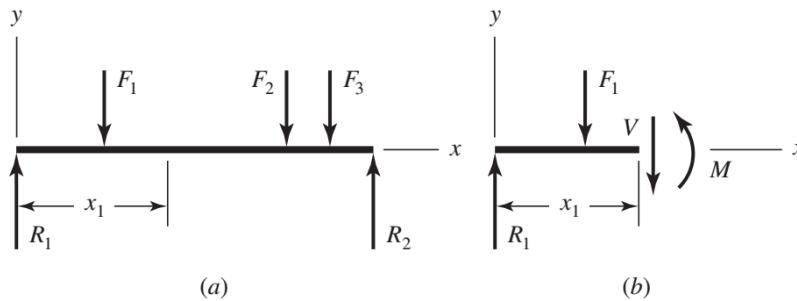
(b) Theoretical and actual dynamic torque at crank pivot



# Beam Loading

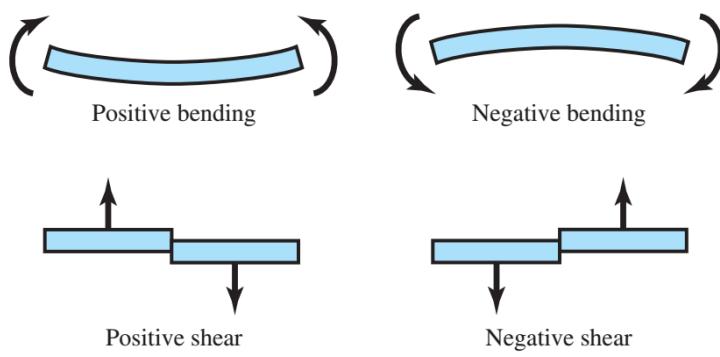
**Figure 3-2**

Free-body diagram of simply-supported beam with  $V$  and  $M$  shown in positive directions (established by the conventions shown in Fig. 3-3).



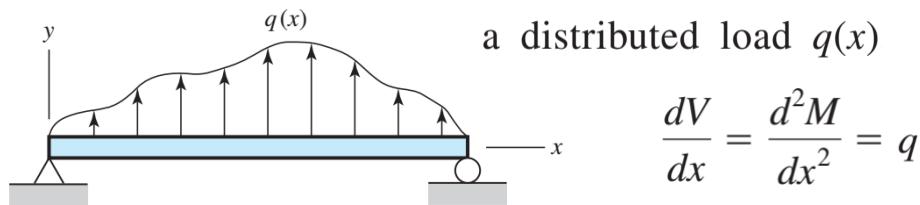
**Figure 3-3**

Sign conventions for bending and shear.



**Figure 3-4**

Distributed load on beam.



$$\frac{dV}{dx} = \frac{d^2M}{dx^2} = q$$

If the beam is cut at some section located at  $x = x_1$  and the left-hand portion is removed as a free body, an internal shear force  $V$  and bending moment  $M$  must act on the cut surface to ensure equilibrium.

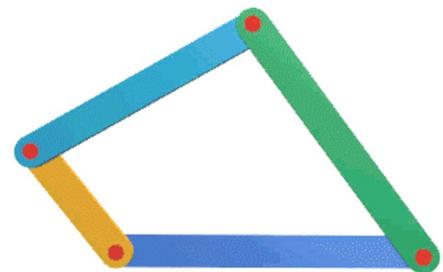
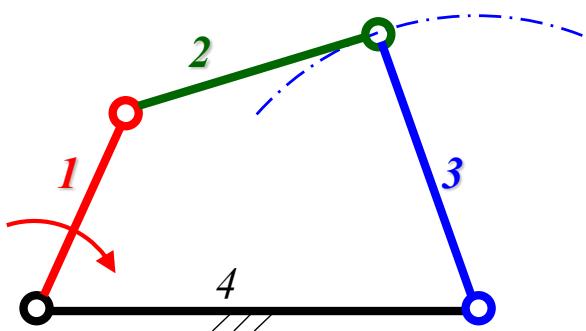
$$V = \frac{dM}{dx}$$

$$\int_{V_A}^{V_B} dV = V_B - V_A = \int_{x_A}^{x_B} q \, dx$$

$$\int_{M_A}^{M_B} dM = M_B - M_A = \int_{x_A}^{x_B} V \, dx$$

*Advanced Textbooks  
are recommended to  
further study this  
topic in depth.*

# 平面四杆机构的基本形式、演变及其应用



## 曲柄摇杆机构

- 在平面四杆机构的两连架杆中
  - 若一个为曲柄，而另一个为摇杆

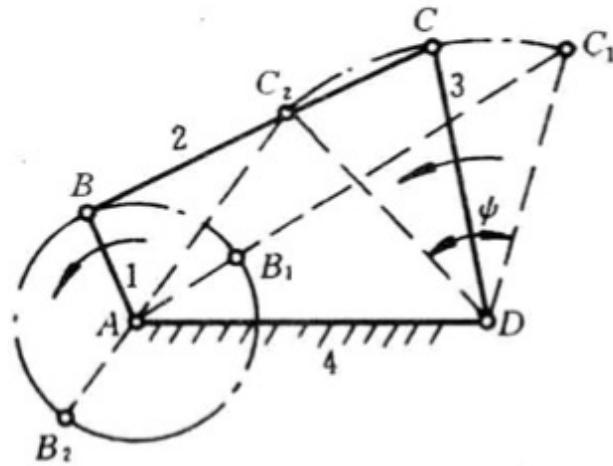
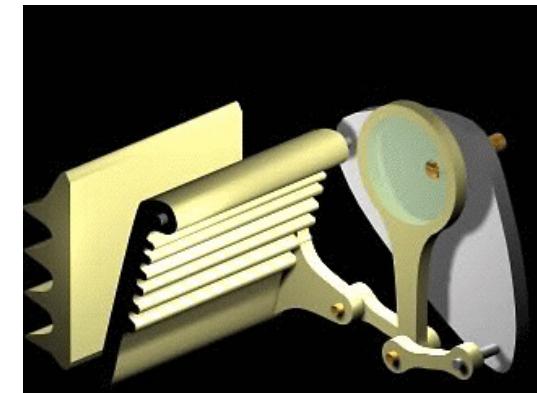


图 2-1 曲柄摇杆机构

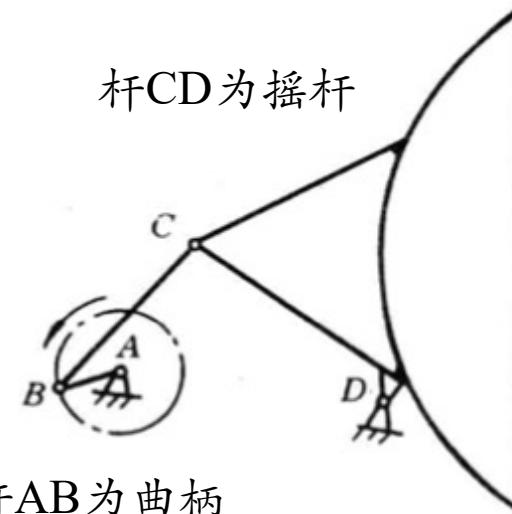
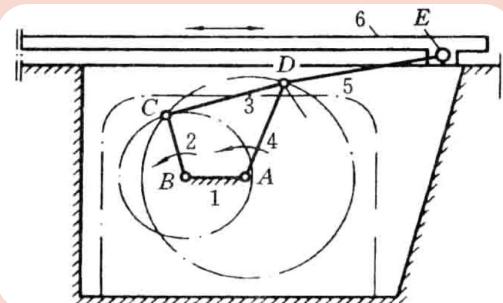


图 2-2 雷达天线机构

利用了曲柄摇杆机构调节天线的俯仰角

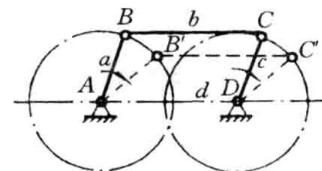
# 双曲柄机构

- 若平面四杆机构的两连架杆均为曲柄

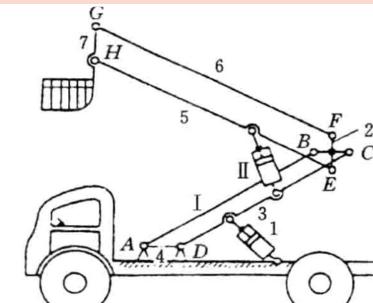


惯性筛双曲柄机构

当曲柄 2 等速回转时，另一曲柄 4 作变速回转，使筛子具有所需的加速度，再利用加速度所产生的惯性力，使大小不同的颗粒在筛上作往复运动，从而达到筛选的目的。



平行四边形机构



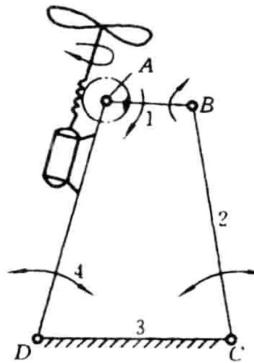
摄影车升降机构

在双曲柄机构中，若两组对边的构件长度相等，则可得到平行四边形机构，由于这种机构两连架杆的运动完全相同，故连杆始终作平动。

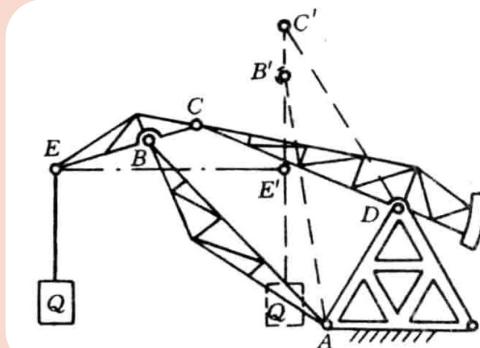
利用平行四边形机构的连杆始终作平动的特点，使与连杆固结在一起的座椅始终保持水平位置，其升降高度的变化也是通过采用两套平行四边形机构来实现的。

# 双摇杆机构

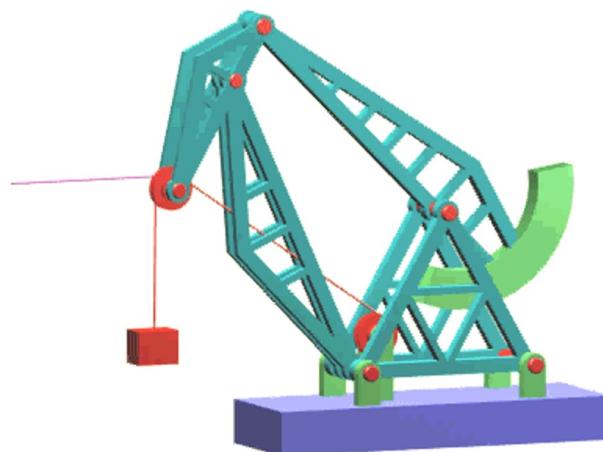
- 在平面四杆机构的两连架杆中
  - 若两连架杆均为摇杆



摇头风扇传动机构



鹤式起重机

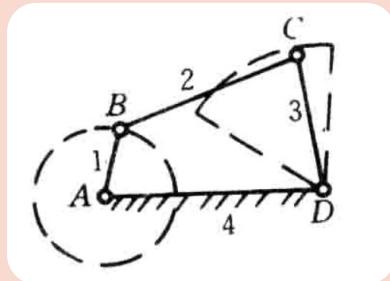


电动机安装在摇杆4上，铰链A处装有一个与连杆1固连成一体的蜗轮，并与电动机轴上的蜗杆相啮合；电动机转动时，通过蜗杆和蜗轮迫使连杆1绕点A作整周转动，从而使连架杆2和4往复摆动，实现风扇摇头的目的

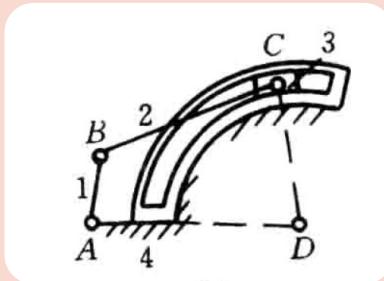
当摇杆AB摆动时，另一摇杆CD随之摆动，可使吊在连杆上E处的重物Q能沿近似水平直线移动

# 平面四杆机构的演变

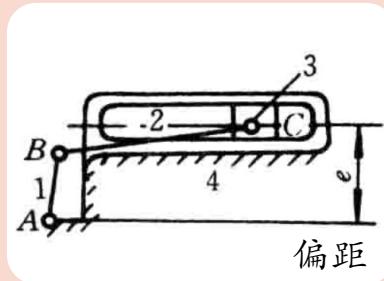
- 转动副转化成移动副
  - 含滑块的平面四杆机构均可看成由铰链四杆机构演变而成



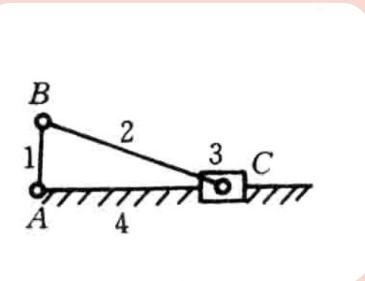
摇杆 3 上点 C 的运动轨迹是以点 D 为圆心，以摇杆长度  $l_{CD}$  为半径所作的圆弧



若改为如图所示的形式，则机构运动的特性完全一样



偏置滑块机构



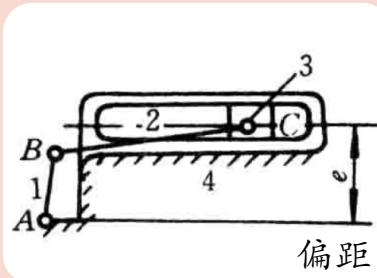
对心滑块机构

若此弧形槽的半径增至无穷大（即点 D 在无穷远处），则弧形槽变成直槽，转动副也就转化成移动副，此时构件 3 也就由摇杆变成了滑块

若滑块上的转动副中心的移动方位线通过曲柄的回转中心，称这种滑块机构为对心滑块机构

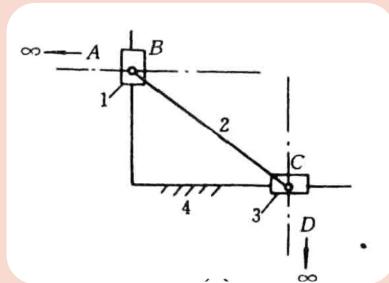
# 平面四杆机构的演变

- 含有两个移动副的四杆机构

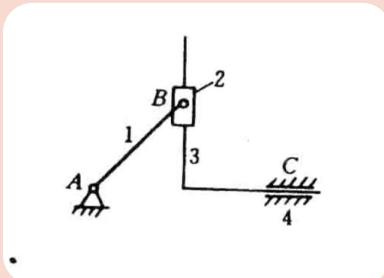


偏置滑块机构

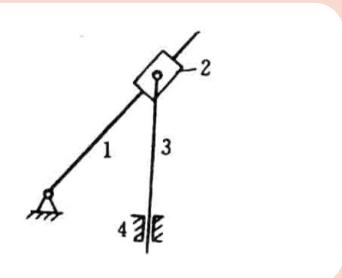
若此弧形槽的半径增至无穷大（即点D在无穷远处），则弧形槽变成直槽，转动副也就转化成移动副，此时构件3也就由摇杆变成了滑块



如将点A移至无穷远处，则转动副A演变成移动副，得到双滑块机构



也可将构件2与构件3之间的转动副C变成移动副，得到曲柄移动导杆机构（又称正弦机构）



若将转动副B变成移动副，则可得到正切机构

# 取不同构件为机架

## • 机构的倒置

- 低副机构具有运动可逆性
- 无论哪一个构件为机架，机构中各构件间的相对运动不变，但选取不同构件为机架时，可得到不同形式的机构

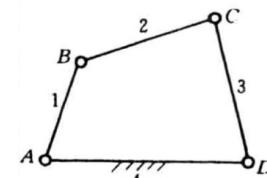
## 曲柄摇杆机构

双曲柄机构

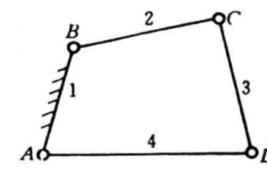
曲柄摇杆机构

双摇杆机构

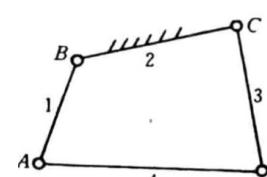
I . 铰链四杆机构



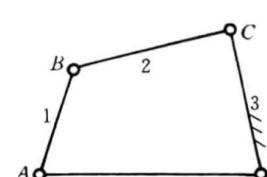
(a) 曲柄摇杆机构



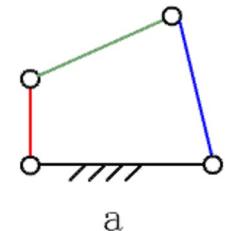
(b) 双曲柄机构



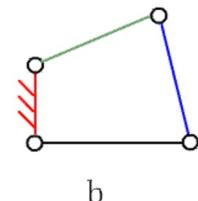
(c) 曲柄摇杆机构



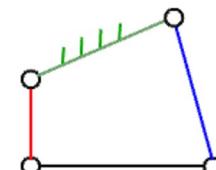
(d) 双摇杆机构



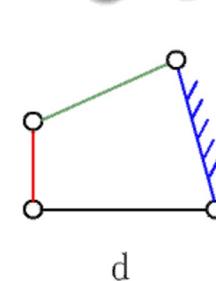
a



b



c



d

## 取不同构件为机架

### • 机构的倒置

- 低副机构具有运动可逆性
- 无论哪一个构件为机架，机构中各构件间的相对运动不变，但选取不同构件为机架时，可得到不同形式的机构

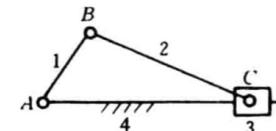
### 曲柄滑块机构

曲柄转动导杆机构

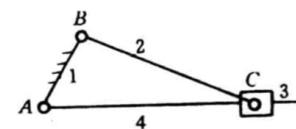
曲柄摇块机构

定块机构

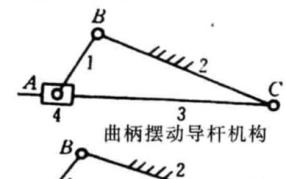
I. 含有一个移动副的平面四杆机构



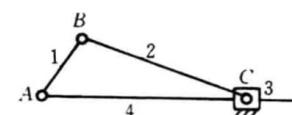
(a) 曲柄滑块机构



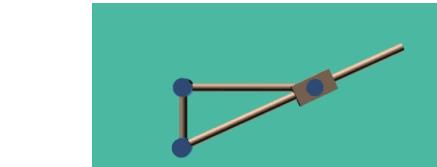
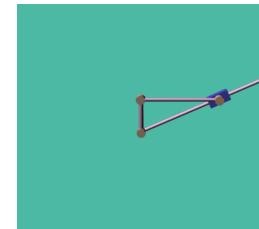
(b) 曲柄转动导杆机构



(c) 曲柄摆动导杆机构



(d) 定块机构



# 取不同构件为机架

- 机构的倒置

- 低副机构具有运动可逆性
- 无论哪一个构件为机架，机构中各构件间的相对运动不变，但选取不同构件为机架时，可得到不同形式的机构

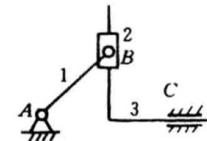
## 曲柄移动导杆机构

双转块机构

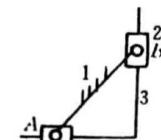
双滑块机构

摆动导杆滑块机构

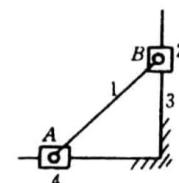
III. 含有两个移动副的平面四杆机构



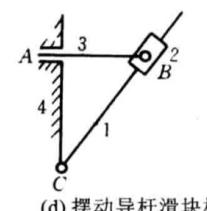
(a) 曲柄移动导杆机构



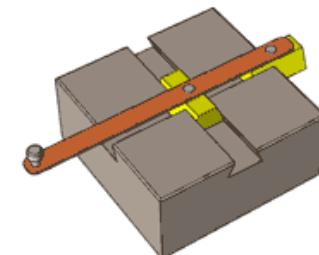
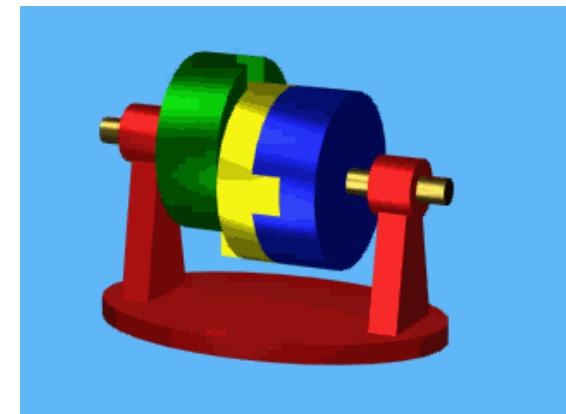
(b) 双转块机构



(c) 双滑块机构



(d) 摆动导杆滑块机构



# 取不同构件为机架

## • 机构的倒置

- 低副机构具有运动可逆性
- 无论哪一个构件为机架，机构中各构件间的相对运动不变，但选取不同构件为机架时，可得到不同形式的机构

曲柄摇杆  
机构

双曲柄机构

曲柄摇杆机  
构

双摇杆机构

曲柄滑块  
机构

曲柄转动导  
杆机构

曲柄摇块机  
构

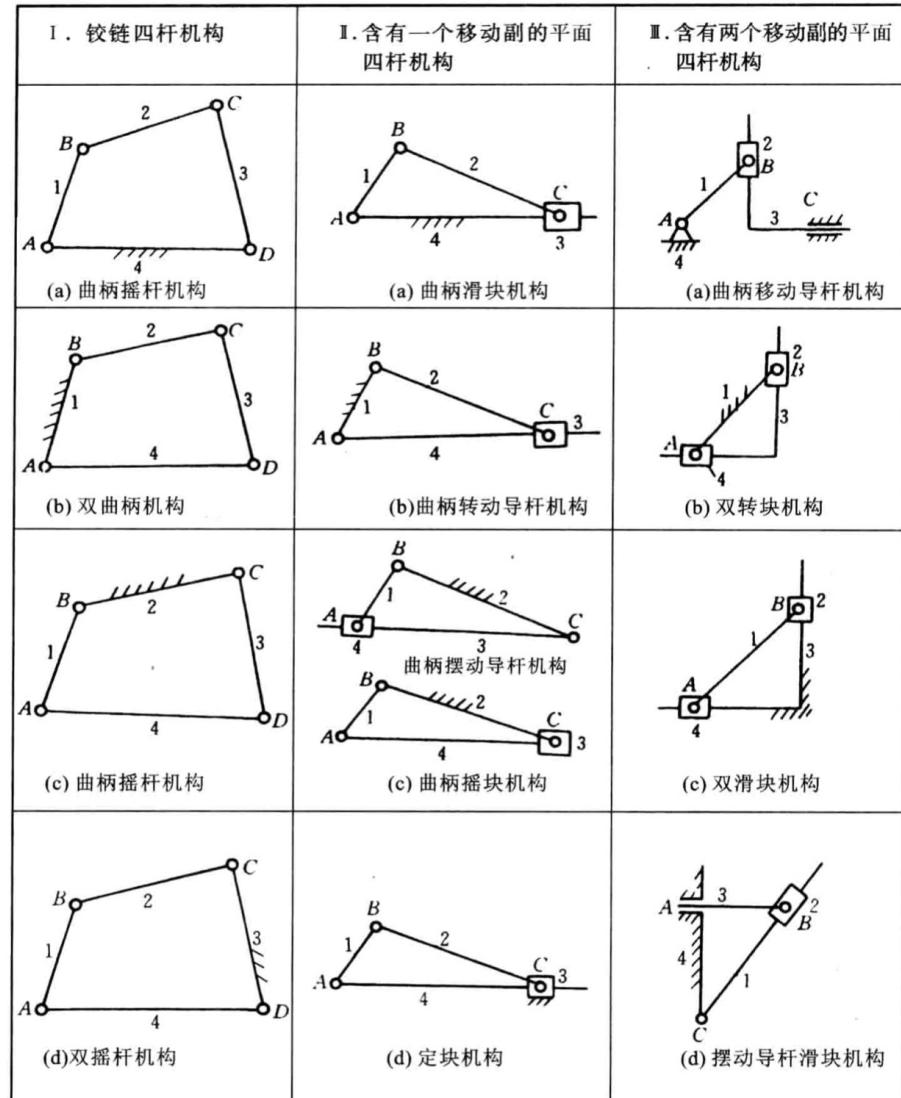
定块机构

曲柄移动  
导杆机构

双转块机构

双滑块机构

摆动导杆滑  
块机构



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# 平面四杆机构设计中的 共性问题

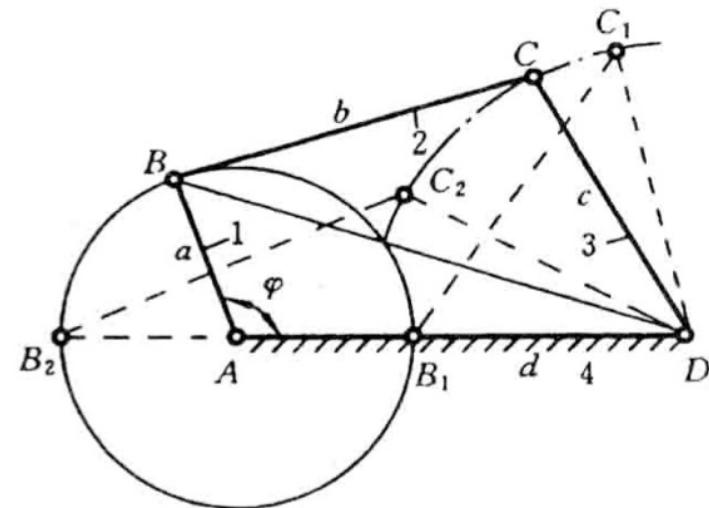
# 平面四杆机构存在曲柄的条件

在工程实际中，  
用于驱动机构运动的原动机通常  
是作整周转动的

要求机构的主动  
件也能作整周转  
动

即希望主动件是  
曲柄

- 设平面四杆机构各杆 1、2、3 和 4 的长度分别为  $a$ 、 $b$ 、 $c$  和  $d$ ，杆 4 为机架，杆 1 和杆 3 为连架杆
- 当  $a < d$  时，只要杆 1 能通过与机架两次共线的位置，则杆 1 必为曲柄
  - $\Delta B_2 C_2 D: a + d \leq b + c$
  - $\Delta B_1 C_1 D: b \leq (d - a) + c$  或  $c \leq (d - a) + b$
  - $a \leq b, a \leq c, a \leq d \Rightarrow$  即杆 AB 为最短杆
- 当  $d < a$  时，同样分析可得
  - $d \leq a, d \leq b, d \leq c \Rightarrow$  即杆 AD 为最短杆

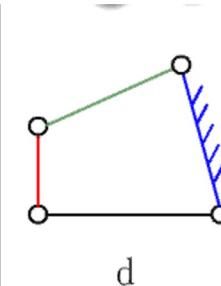
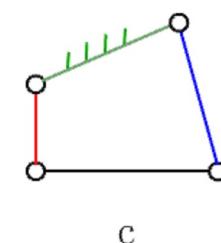
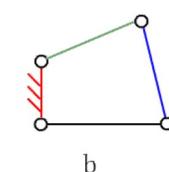
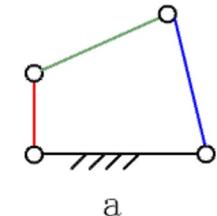
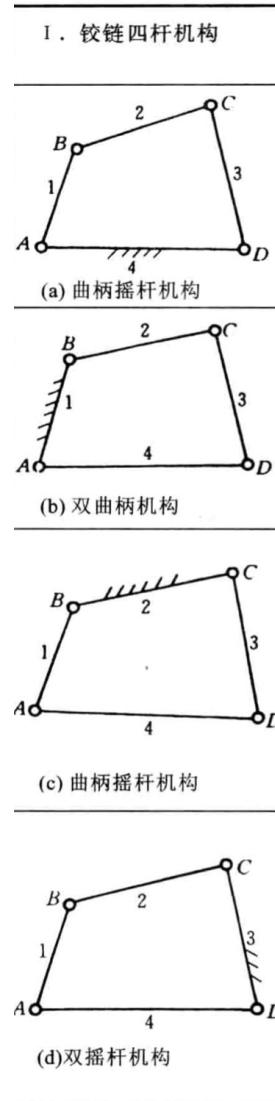


平面四杆机构存在曲柄的条件是：

- (1) 连架杆与机架中必有一杆为平面四杆机构中的**最短杆**
- (2) 最短杆与最长杆的杆长之和应小于或等于其余两杆的**杆长之和**

# 平面四杆机构存在曲柄的条件

- 在平面四杆机构中，如果最短杆与最长杆的长度之和小于或等于其他两杆长度之和，且：
  - ① 曲柄摇杆机构：**以最短杆的相邻构件为机架，则最短杆为曲柄，另一连架杆为摇杆
  - ② 双曲柄机构：**以最短杆为机架，则两连架杆均为曲柄
  - ③ 双摇杆机构：**以最短杆的对边构件为机架，则无曲柄存在





# ME303: 机械设计

## 2022年秋季

谢谢~

宋超阳  
南方科技大学