ME303 Introduction to Mechanical Design

Lecture 10 Power Transmission Case Study

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Agenda

Week 09, Wednesday, Nov 06, 2019

- Design Sequence for Power Transmission
- Power and Torque Requirements
- Gear Specification
- Shaft Layout
- Force Analysis
- Shaft Material Selection
- Shaft Design for Stress | Deflection
- Bearing Selection
- Key and Retaining Ring Selection
- Final Analysis



Introduction

Transmission of power from a source through a machine to an output actuation.

- The design of a system to transmit power requires attention to
 - the design and selection of individual components (gears, bearings, shaft, etc.).
 - these components are not independent.
- An overview of a power transmission system design,
 - Demonstrating how to incorporate the details of each component into an overall design process.

A compound reverted gear train.





Problem Specification

Case Study Part 1

Power to be delivered: 20 hp Input speed: 1750 rpm Output speed: 82-88 rev/min Usually low shock levels, occasional moderate shock Input and output shafts extend 4 in outside gearbox Maximum gearbox size: 14-in × 14-in base, 22-in height Output shaft and input shaft in-line Gear and bearing life > 12 000 hours; infinite shaft life

Design Specifications An Example

A two-stage, compound reverted gear train



Design Sequence for Power Transmission

Understanding the dependencies between the parts of the problem saves time

- Power and torque requirements
 - Overall sizing needs for the entire system
- Gear specification
 - Suitable gear ratios and torque transmission
- Shaft layout
 - General layout, including axial location of gears and bearings
 - How to transmit the torque, no need to size them yet
- Force analysis
 - Free-body, shear force, and bending moment diagram
 - Forces at the bearings can be determined.
- Shaft material selection
 - For fatigue design, select the material first, and then check for satisfactory results.

- Shaft design for stress (fatigue and static)
 - Similar to last chapter, predict critical locations of the shaft, approximate stress concentrations, and estimate for shaft diameters
- Shaft design for deflection
 - All shaft geometry determined, critical deflections at the bearing and gear locations can be checked by analysis
- Bearing selection
 - Gear selection from a catalog with slight adjustment to match specifications
- Key and retaining ring selection
 - Based on standard sizes, little change to the overall design
- Final analysis
 - A complete analysis from start to finish to provide a final check and specific safety factors for the actual system. SUSTech

Power and Torque Requirements

Power transmission systems will typically be specified by a power capacity

- The combination of torque and speed that the unit can endure, i.e. a 40-horsepower gearbox
 - Ideal case: power in equals power out
 - Actual case: small power loss due to factors like friction
 - Practice: power loss can be negligible $(1 \sim 2\%)$, but still depends
- Torque is typically not constant through a transmission system.

gear ratio

 $H = T_i \omega_i = T_o \omega_o \qquad e = \omega_o / \omega_i = T_i / T_o$

• With a constant power, a gear ratio to decrease the angular velocity will simultaneously increase torque.



Gear Specification

With the gear train value known, the next step is to determine appropriate gears.

- As a rough guideline,
 - A train value of up to 10 to 1 can be obtained with one pair of gears.
 - Greater ratios can be obtained by compounding additional pairs of gears.
 - The compound reverted gear train can obtain a train value of up to 100 to 1.
- It is best to design with teeth numbers rather than diameters
 - Numbers of teeth on gears must be integers
 - Care should be taken at this point to find the best combination of teeth numbers to minimize the overall package size.
- Production quantity is another factor of choice



Speed, Torque, and Gear Ratios

Case Study Part 2

Continue the case study by determining appropriate tooth counts to reduce the input speed of $\omega_i = 1750$ rev/min to an output speed within the range 82 rev/min $< \omega_o < 88$ rev/min

Use the notation for gea	numbers from Fig. 18-1. Choose	mean value for initial
design, $\omega_5 = 85$ rev/min.		
	$e = \frac{\omega_5}{\omega_2} = \frac{85}{1750} = \frac{1}{20.59}$	Eq. (18–2)
For a compound reverted	geartrain,	
	$e = \frac{1}{N_2} \frac{N_4}{N_4}$	Eq. (13-30) p 691
	20.59 N ₃ N ₅	

For smallest package size, let both stages be the same reduction. Also, by making the two stages identical, the in-line condition on the input and output shaft will automatically be satisfied.

 $\frac{N_2}{N_3} = \frac{N_4}{N_5} = \sqrt{\frac{1}{20.59}} = \frac{1}{4.54}$

For this ratio, the minimum number of teeth from Eq. (13-11), p. 678, is 16.

$N_2 = N_4 = 16$ teeth	
$N_3 = 4.54(N_2) = 72.64$	

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Power to be delivered: 20 hp Input speed: 1750 rpm Output speed: 82-88 rev/min Usually low shock levels, occasional moderate shock Input and output shafts extend 4 in outside gearbox Maximum gearbox size: 14-in × 14-in base, 22-in height Output shaft and input shaft in-line Gear and bearing life > 12 000 hours; infinite shaft life





Acceptable



Speed, Torque, and Gear Ratios

Case Study Part 2

Once final tooth counts are specified, determine values of

(a) Speeds for the intermediate and output shafts

(b) Torques for the input, intermediate and output shafts, to transmit 20 hp.

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$$H = T_2 \omega_2 = T_5 \omega_5$$

$$T_2 = H/\omega_2 = \left(\frac{20 \text{ hp}}{1750 \text{ rev/min}}\right) \left(550 \frac{\text{ft-lbf/s}}{\text{hp}}\right) \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) \left(60 \frac{\text{s}}{\text{min}}\right)$$

$$T_2 = 60.0 \text{ lbf} \cdot \text{ft}$$





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Estimate the Minimum Diametral Pitch

by writing an expression for the gearbox size in terms of gear diameters

• The overall height of the gearbox

 $Y = d_3 + d_2/2 + d_5/2 + 2/P + \text{clearances} + \text{ wall thicknesses}$ $d_i = N_i/P$ addendum height of the teeth on gears 3 and 5 that extend beyond the pitch diameters

 $Y = N_3/P + N_2/(2P) + N_5/(2P) + 2/P + \text{clearances} + \text{wall thicknesses}$

 $P = (N_3 + N_2/2 + N_5/2 + 2)/(Y - \text{clearances} - \text{wall thicknesses})$

This is the minimum value that can be used for diametral pitch, and therefore the maximum tooth size, to stay within the overall gearbox constraint.

• It should be rounded *up* to the next standard diametral pitch, which reduces the maximum tooth size.



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Next Steps

AGMA Approach

- Determine suitable gear parameters
 - Based on Bending and Contact Stress
 - Primary design parameters: material, diametral pitch, face width.
- Diametral pitch => Gear Diameters / Pitch-line Velocities / Transmitted Loads
 - Or check gear catalogs to find available face widths for the diametral pitch and number of teeth.
- Material Choice => Safety Factors
 - Analyze the most critical gear first, usually the smaller one on the high-torque (low-speed) end of the gear box
 - Material treatment to widen the available choices
- An iteration process until acceptable results are obtained.



Gear Specification

Gear Study Part 3



Continue the case study by specifying appropriate gears, including pitch diameter, diametral pitch, face width, and material. Achieve safety factors of at least 1.2 for wear and bending.

Estimate the minimum diametral pitch for overall gearbox height = 22 in.





Check Critical Gear for Safety Factor

Start with gear 4, since it is the smallest gear, transmitting the largest load. It will likely be critical. Start with wear by contact stress, since it is often the limiting factor.



Since gear specifications are readily available on the Internet, we might as well check for commonly available face widths.

On www.globalspec.com, entering P = 6 teeth/in and d = 2.67 in, stock spur gears from several sources have face widths of 1.5 in or 2.0 in. These are also available for the meshing gear 5 with d = 12 in.

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Gear 4 Wear

Gear 4 => Gear 5 => Gear 2 => Gear 3



Gear 4 Bending



Fig. 14-6, p. 745

Everything else is the same as before.



J = 0.27

 $K_B = 1$

Using Grade 2 carburized and hardened, same as chosen for wear, find $S_t = 65\ 000\ psi$ (Table 14-3, p. 740). $\sigma_{all} = S_t Y_N = 58\ 500\ psi$

The factor of safety for bending of gear 4 is $n = \frac{\sigma_{\text{all}}}{\sigma} = \frac{58\ 500}{38\ 570} = 1.52$





Gear 5 Bending & Wear

Gear 4 => Gear 5 => Gear 2 => Gear 3







Gear 2 Wear & Bending

Gear 4 => Gear 5 => Gear 2 => Gear 3



Gear 3 Bending & Wear

Gear 4 => Gear 5 => Gear 2 => Gear 3



Shaft Layout

Case Study Part 4

The gear widths are known at this point.

Bearing widths are guessed, allowing a little more space for larger bearings on the intermediate shaft where bending moments will be greater. Continue the case study by preparing a sketch of the gearbox sufficient to determine the axial dimensions. In particular, estimate the overall length, and the distance between the gears of the intermediate shaft, in order to fit with the mounting requirements of the other shafts.



Force Analysis, Material Selection, Design for Stress & Deflection

Case Study Parts 5 & 6

Please refer to previous lecture for details of this example.

Solution Free body Diagram => Torque => Shear-Moment Diagram => Endurance Strength => Shaft Diameter

Generate shear-moment enalysis to get ne et the bearings. diagrams for two planes. $R_{A_0} = 115.0$ lbf $R_{A_0} = 356.7$ lbf Re. = 1776.0 lbf Start with point I, where the bending moment is high, there is a stress concentration at the shoulder and the torque is present At I, $M_a = 3651$ lbf \cdot in $T_m = 3240 \text{ lbf} \cdot \text{in},$ $M_m = T_a = 0$ Combine orthogonal planes as vectors to get total moments, e.g., at J, $\sqrt{3996^2 + 1632^2} =$ e.g., at J, √ 4316 lbf • in. AncoraSIR.con

Quickly check if point M might be critical. Only bending is present, and the moment is small, but the diameter is small and the stress concentration is high for a sharp fillet required for a bearing. From the moment diagram, $M_a = 959$ lbf · in, and $M_m = T_m = T_a = 0$. Estimate $K_c = 2.7$ from Table 7-1, d = 1.0 in, and fillet radius r to fit a

typical bearing. r/d = 0.02, r = 0.02(1) = 0.02

1	
q = 0.7 (Fig	. 6–20)
$K_f = 1 + (0.7)$	7)(2.7 - 1) = 2.19
$32K_f M_a$	32(2.19)(959)
$\sigma_a = \frac{\pi d^3}{\pi d^3}$	$=\frac{1}{\pi(1)^3}$ = 21 390 psi
0 0	
$n_c = \frac{S_c}{S_c} = \frac{3}{2}$	$\frac{5300}{156} = 1.56$
$\sigma_a = 21$	390

Should be OK. Close enough to recheck after bearing is selected.

With the diameters specified for the critical locations, fill in trial values for the rest of the diameters, taking into account typical shoulder heights for bearing and gear support. $D_1 = D_7 = 1.0 \text{ in}$ $D_2 = D_6 = 1.4 \text{ in}$ $D_3 = D_5 = 1.625 \text{ in}$ $D_4 = 2.0 \text{ in}$

Bearing A Gear 3 d ₃ = 12		Gear 4 d ₄ = 2.67	Bearing B
	↓ Da	+	D ₅ D ₇
	g	9 8	510 KL M B N

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Bearing Selection

Case Study Part 7

Continue the case study by selecting appropriate bearings for the intermediate shaft, with a reliability of 99 percent. The problem specifies a design life of 12000 h. The intermediate shaft speed is 389 rev/min. The estimated bore size is 1 in, and the estimated bearing width is 1 in

	6000	11	1	610000	
At the shaft speed	of 389 rev	/min the (desian lite o	of 12 000 h	correlates to a
				•	
bearing life of L_D	$= (12 \ 000)$	h)(60 min/	/h)(389 rev/1	$\min) = 2.8$	$3 \times 10^{\circ}$ rev.

	Sto	art	t w	ith b	bea	rin	g E	3 si	nc	e i	t h	as	t)	ne	hig	ghe	r l	oad	s (anc	v k	vill	lik	ely	r	ais	e a	ny		
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		M	, 				$\lfloor 0$.02	2 +	- (-	4.4	59	-	- 0	.02	2)(1 -	- 0	.99	9)1	/1.	483]							

A check on the Internet for available bearings (www.globalspec.com is one good starting place) shows that this load is relatively high for a ball bearing with bore size in the neighborhood of 1 in. Try a cylindrical roller bearing. Recalculating F_{RB} with the exponent a = 10/3 for roller bearings, we obtain

 $F_{RB} = 16\ 400\ \text{lbf}$

at the bearings. $R_{Az} = 115.0 \text{ lbf}$ $R_{Ay} = 356.7 \text{ lbf}$ $R_{Bz} = 1776.0 \text{ lbf}$ $R_{By} = 725.3 \text{ lbf}$ w_{23}''

Perform free body diagram

analysis to get reaction forces





Bearing Selection

Case Study Part 7

Cylindrical roller bearings are available from several sources in this range. A specific one is chosen from SKF, a common supplier of bearings, with the following specifications:

Cylindrical roller bearing at right end of shaft $C = 18\ 658\ 1bf$, ID = 1.181 1 in, OD = 2.834 6 in, W = 1.063 in Shoulder diameter range = 1.45 in to 1.53 in, and maximum fillet radius = 0.043 in

For bearin	g A, again	assuming a ball bearing,	
		$[2.8 \times 10^8 / 10^6]^{1/3}$	
	$F_{RA} = 1$	375 = 4070 lbf	
		$\lfloor 0.02 + 4.439(1 - 0.99)^{1/1.405} \rfloor$	

A specific ball bearing is chosen from the SKF Internet catalog.

Deep-groove ball bearing at left end of shaft

C = 5058 lbf, ID = 1.000 in, OD = 2.500 in, W = 0.75 in

Shoulder diameter range = 1.3 in to 1.4 in, and maximum fillet radius = 0.08 in



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Continue the case study by specifying appropriate keys for the two Key Design gears on the intermediate shaft to provide a factor of safety of 2. The gears are to be custom bored and keyed to the required specifications. Case Study Part 8 Transmitted torque: T = 3240 lbf-in Bore diameters: $d_3 = -d_4 = -1.625$ in Gear hub lengths: $l_3 = 1.5$ in, $l_4 = 2.0$ in From Table 7-6, p. 383, for a shaft diameter of 1.625 in, choose a square key with side dimension $t = \frac{3}{8}$ in. Choose 1020 CD material, with $S_v = 57$ kpsi. The force on the key at the surface of the shaft is $F = \frac{T}{r} = \frac{3240}{1.625/2} = 3988$ lbf Checking for failure by crushing, we find the area of one-half the face of the key is used. $n = \frac{S_y}{\sigma} = \frac{S_y}{F/(tl/2)}$ Solving for *l* gives $l = \frac{2Fn}{tS} = \frac{2(3988)(2)}{(0.375)(57000)} = 0.75 \text{ in}$ Since both gears have the same bore diameter and transmit the same torque, the same key specification can be used for both. AncoraSIR.com

Final Analysis

Remember that analysis is much more straightforward than design, so the investment of time for the final analysis will be relatively small.



shows the important dimensions and dimensional tolerances in a form that is generally considered satisfactory for small production quantities where direct attention is given to manufacturing methods.

	NOTES	UNLESS OTHERWISE SPECIFIED:	"™ Intermediate Shaft					
	l. Material: 1050 CD	DIMENSIONS ARE IN INCHES	COMPANY					
	2. Fillet radii for all grooves: R 0.008	.×× ± 0.010 .××× ± 0.005	^{51ZE} A4 ^{DWG ND.} 012 - 001					
AncoraSIR.com	- 0.01		Scale 1:1 Sheet 1 of 1					

Fabrication Update Report

- Submission Due Nov 10 noon
- Online at course website

Next class

- Lab for Group 1: Design Consultation
- Friday 0800-1000, Nov 01
- Room 412, 5 Wisdom Valley
- **Discussion for Group 2**: Design Consultation
- Friday 0800-1000, Nov 01
- Room 202, 1 Lychee Park

Thank you!

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