ME303 Introduction to Mechanical Design

[Lecture 0](mailto:songcy@sustech.edu.cn)9 Shafts & Shaft Compone

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Agenda

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- Introduction
- Shaft Materials
- Shaft Layout
- Example of Shaft Design
- Shaft Components
- Limits and Fits

About the Shaft

A rotating member, usually of circular cross section, used to transmit power or motion.

- A complete shaft design has much interdependence on the design of the components.
- The geometry of the entire shaft is not needed for shaft sizing.
	- A **stress analy**sis at a specific point on a shaft can be made using only the shaft geometry in the vicinity of that point.
	- Locate the critical areas, **size these** to meet the strength requirements, and then **size the rest of the shaft** to meet the requirements of the shaft-supported elements.
- The **deflection and slope analyses** cannot be made until the geometry of the entire shaft has been defined.
	- Thus deflection is a function of the geometry *everywhere*,
- AncoraSIR.com • whereas the stress at a section of interest is a function of *local* geometry.

Material Selection

Deflection => Stiffness => Modulus of Elasticity => Geometric Decisions

Strength => Resist Loading Stress => Material Selection & Treatment

- AISI 1020-1050 steels
	- Low carbon, cold-drawn or hot-rolled steel
- AISI 1340-50, 3140-50, 4140, 4340, 5140, and 8650.
	- Significant strengthening from heat treatment and high alloy content are often not warranted.
	- Typical alloy steels for heat treatment
		- Start with an inexpensive, low or medium carbon steel for the first time through the design calculations
		- If strength considerations turn out to dominate over deflection, then a higher strength material should be tried, allowing the shaft sizes to be reduced until excess deflection becomes an issue.
		- The cost of the material and its processing must be weighed against the need for smaller shaft diameters.
- AISI 1020, 4320, 4820, and 8620 for surface hardening
	- Shafts usually don't need to be surface hardened unless they serve as the actual journal of a bearing surface.
- The amount to be produced is a salient factor..
	- For low production, turning is the usual primary shaping process. An economic viewpoint may require removing the least material.
	- High production may permit a volume conservative shaping method (hot or cold forming, casting), and minimum material in the shaft can become a design goal. Cast iron may be specified if the production quantity is high, and the gears are to be integrally cast with the shaft.
- Properties of the shaft locally depend on its history
	- Cold work, cold forming, rolling of fillet features, heat treatment, including quenching medium, agitation, and tempering regimen

Geometric Layout

The general layout of a shaft to accommodate shaft elements must be specified early in the design process in order to perform a free body force analysis and to obtain shear-moment diagrams.

- The geometry of a shaft
	- Generally a stepped cylinder.
- The shaft geometric configuration
	- Often simply a revision of existing models in which a limited number of changes must be made.
- There are no absolute rules for specifying the general layout.
	- Axial Layout of Components
	- Supporting Axial Loads
	- Providing for Torque Transmission
	- Assembly and Disassembly

Shaft Design for Stress: Critical Locations

A few potentially critical locations will suffice the evaluation of stress in shaft design.

- Common places of Critical Locations
	- Usually on the outer surface
	- At the axial locations where the bending moment is large
	- Where the torque is present
	- And where stress concentration exist
- Most shafts will transmit torque through a portion of the shaft.
- The bending moments on a shaft can be determined by *shear and bending moment diagrams*.
	- In two planes: radial and tangential
- Axial stresses on shafts are usually negligibly small
	- When using the axial components transmitting through helical gears or tapered roller bearings
- AncoraSIR.com • When comparing to the bending moment stress

Shaft Design for Stress: Shaft Stresses

Bending, torsion, and axial stresses may be present in both midrange and alternating components.

- For analysis, it is simple enough to combine the different types of stresses into *alternating* and *midrange* **von Mises stresses**
- The fluctuating stresses due to bending and torsion, assuming a solid shaft with round cross section.

$$
\sigma_a = K_f \frac{32M_a}{\pi d^3} \qquad \sigma_m = K_f \frac{32M_m}{\pi d^3}
$$

T: Torques *m*: midrange *a*: alternating

M: bending moments

$$
\tau_a = K_{fs} \frac{16T_a}{\pi d^3} \qquad \tau_m = K_{fs} \frac{16T_m}{\pi d^3}
$$

 K_f : fatigue stress-concentration factor for bending K_{fs} : fatigue stress-concentration factor for torsion

• For rotating round solid shafts, neglecting axial loads, the fluctuating von Mises stresses are given by

$$
\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[\left(\frac{32K_fM_a}{\pi d^3} \right)^2 + 3\left(\frac{16K_{fs}T_a}{\pi d^3} \right)^2 \right]^{1/2}
$$

$$
\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[\left(\frac{32K_fM_m}{\pi d^3} \right)^2 + 3\left(\frac{16K_{fs}T_m}{\pi d^3} \right)^2 \right]^{1/2}
$$

Shaft Design for Stress: Shaft Diameter

Can be evaluated using an appropriate failure curve on the modified Goodman diagram.

• the fatigue failure criteria for the modified Goodman line as expressed previously

$$
\frac{1}{n} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} \qquad \qquad \frac{1}{n} = \frac{16}{\pi d^3} \left\{ \frac{1}{S_e} [4(K_f M_a)^2 + 3(K_{fs} T_a)^2]^{1/2} + \frac{1}{S_{ut}} [4(K_f M_m)^2 + 3(K_{fs} T_m)^2]^{1/2} \right\} \n d = \left(\frac{16n}{\pi} \left\{ \frac{1}{S_e} [4(K_f M_a)^2 + 3(K_{fs} T_a)^2]^{1/2} + \frac{1}{S_{ut}} [4(K_f M_m)^2 + 3(K_{fs} T_m)^2]^{1/2} \right\} \right)^{1/3}
$$

- Similar expressions can be obtained for any of the common failure criteria by substituting the von Mises stresses into any of the failure criteria expressed
	- DE-Goodman | DE-Gerber | DE-ASME Elliptic | DE-Soderberg
- For a rotating shaft with constant bending and torsion,
	- Setting M_m and T_a equal to 0, which simply drops out some of the terms.
- It is necessary to consider the possibility $\sigma'_{max} = [(\sigma_m + \sigma_a)^2 + 3(\tau_m + \tau_a)^2]^{1/2}$ $n_y = \frac{S_y}{\sigma'_{max}}$ of static failure in the first load cycle. $= \left[\left(\frac{32K_f(M_m+M_a)}{\pi d^3} \right)^2 + 3 \left(\frac{16K_{fs}(T_m+T_a)}{\pi d^3} \right)^2 \right]^{1/2}$
	- Von Mises maximum stress & Yield strength

Shaft Design for Stress: Estimating Stress Concentrations

The stress analysis process for fatigue is highly dependent on stress concentrations.

- *D/d* to be between 1.2 and 1.5
- r/d typically ranging from around 0.02 to 0.06

First Iteration Estimates for Stress-Concentration Factors K_t and K_t .

Warning: These factors are only estimates for use when actual dimensions are not yet determined. Do *not* use these once actual dimensions are available.

Missing values in the table are not readily available.

Techniques for reducing stress concentration at a shoulder supporting a bearing with a sharp radius.

- a) Large radius undercut into the shoulder.
- b) Large radius relief groove into the back of the shoulder.
- c) Large radius relief groove into the small diameter.

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Shaft Design Example

A double reduction gearbox design has developed to the point that the general layout and axial dimensions of the countershaft carrying two spur gears has been proposed, as shown in Fig. 7-10. The gears and bearings are located and supported by shoulders, and held in place by retaining rings. The gears transmit torque through keys. Gears have been specified as shown, allowing the tangential and radial forces transmitted through the gears to the shaft to be determined as follows.

$$
W_{23}^t = 540 \text{ lbf}
$$

\n
$$
W_{54}^t = 2431 \text{ lbf}
$$

\n
$$
W_{23}^r = 197 \text{ lbf}
$$

\n
$$
W_{54}^r = 885 \text{ lbf}
$$

where the superscripts t and r represent tangential and radial directions, respectively; and, the subscripts 23 and 54 represent the forces exerted by gears 2 and 5 (not shown) on gears 3 and 4, respectively.

Proceed with the next phase of the design, in which a suitable material is selected, and appropriate diameters for each section of the shaft are estimated, based on providing sufficient fatigue and static stress capacity for infinite life of the shaft, with minimum safety factors of 1.5.

Solution

Free body Diagram => Torque => Shear-Moment Diagram => Endurance Strength => Shaft Diameter

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Conservative 1st Pass

Trail & Error to find the optimal dimension

• Assume generous fillet radius for gear at I.

From Table 7-1, estimate $K_t = 1.7, K_s = 1.5$. For quick, conservative first pass, assume $K_f = K_t, K_{fs} = K_{ts}$. Choose inexpensive steel, 1020 CD, with $S_{ut} = 68$ kpsi. For S_e , $k_a = aS_{ut}^b = 2.7(68)^{-0.265} = 0.883$ Eq. $(6-19)$ Guess $k_b = 0.9$. Check later when d is known. $k_c = k_d = k_e = 1$ $S_e = (0.883)(0.9)(0.5)(68) = 27.0$ kpsi Eq. $(6-18)$

First Iteration Estimates for Stress-Concentration Factors K , and K . Warning: These factors are only estimates for use when actual dimensions are not yet determined. Do *not* use these once actual dimensions are available

Missing values in the table are not readily available.

- For first estimate of the small diameter at the shoulder at point I, use the DE-Goodman criterion.
	- This criterion is good for the and conservative

Also check this diameter at the end of the keyway, just to the right of point *I*, and at the groove at point *K*. From moment diagram, estimate *M* at

Assume the radius at the bottom of the keyway will be end of keyway to be $M = 3750$ lbf-in. the standard $r = 0.02d = 0.02(1.625) = 0.0325$ in.

The keyway turns out to be more critical than the shoulder. We can either increase the diameter or use a higher strength material. Unless the deflection analysis shows a need for larger diameters, let us choose to increase the strength. We started with a very low strength and can afford to increase it some to avoid larger sizes

Try 1050 CD with $S_{ut} = 100$ kpsi.

This satisfies the goal for the design factor to be at least 1.5

Check at the groove at K , since K_t for flat-bottomed grooves are often very high. From the torque diagram, note that no torque is present at the groove. From the moment diagram, $M_a = 2398$ lbf \cdot in, $M_m = T_a = T_m = 0$. To quickly check if this location is potentially critical, just use $K_f = K_t = 5.0$ as an estimate, from Table 7-1.

This is low. We will look up data for a specific retaining ring to obtain K_{ϵ} more accurately. With a quick online search of a retaining ring specification using the website www.globalspec.com, appropriate groove specifications for a retaining ring for a shaft diameter of 1.625 in are obtained as follows: width, $a = 0.068$ in; depth, $t = 0.048$ in; and corner radius at bottom of groove, $r = 0.01$ in. From Fig. A-15-16, with $r/t = 0.01/0.048 + 0.208$, and $a/t = 0.068/0.048 = 1.42$

$$
K_t = 4.3, q = 0.65 \text{ (Fig. 6-20)}
$$
\n
$$
K_f = 1 + 0.65(4.3 - 1) = 3.15
$$
\n
$$
\sigma_a = \frac{32K_fM_a}{\pi d^3} = \frac{32(3.15)(2398)}{\pi (1.625)^3} = 17930 \text{ psi}
$$
\n
$$
n_f = \frac{S_e}{\sigma_a} = \frac{33300}{17930} = 1.86
$$

Quickly check if point M might be critical. Only bending is present, and the moment is small, but the diameter is small and the stress concentration is First Iteration Estimates for Stress-Concentration Factors K , and K . high for a sharp fillet required for a bearing. From the moment diagram, $M_a = 959$ lbf \cdot in, and $M_m = T_m = T_a = 0$. Estimate $K_t = 2.7$ from Table 7-1, $d = 1.0$ in, and fillet radius r to fit a typical bearing.

 $r/d = 0.02, r = 0.02(1) = 0.02$ $q = 0.7$ (Fig. 6–20) $K_f = 1 + (0.7)(2.7 + 1) = 2.19$ $\sigma_a = \frac{32K_fM_a}{\pi d^3} = \frac{32(2.19)(959)}{\pi (1)^3} = 21390 \text{ psi}$ $n_f = \frac{S_e}{\sigma_a} = \frac{33300}{21390} = 1.56$

Should be OK. Close enough to recheck after bearing is selected.

With the diameters specified for the critical locations, fill in trial values for the rest of the diameters, taking into account typical shoulder heights for bearing and gear support.

Warning: These factors are only estimates for use when actual dimensions are not ye determined. Do not use these once actual dimensions are available.

Missing values in the table are not readily available

 $D_1 = D_7 = 1.0$ in $D_2 = D_6 = 1.4$ in $D_3 = D_5 = 1.625$ in $D_4 = 2.0$ in

Shaft Components: Setscrews

the setscrew depends on compression to develop the clamping force

- The resistance to axial motion of the collar or hub relative to the shaft is called *holding power*
	- due to frictional resistance of the contacting portions of the collar and shaft as well as any slight penetration of the setscrew into the shaft.

Shaft Components: Keys and Pins

used on shafts to secure rotating elements, such as gears, pulleys, or other wheels.

• *Keys* are used to enable the transmission of torque from the shaft to the shaftsupported element.

• *Pins* are used for axial positioning and for the transfer of torque or thrust or both.SUS Leck

Shaft Components: Retaining Rings

Axially position a component on a shaft or in a housing bore

Care should be taken in using retaining rings, particularly in locations with high bending stresses

Limits and Fits

capital letters always refer to the hole; lowercase letters are used for the shaft

- *Basic size* is the size to which limits or deviations are assigned and is the same for both members of the fit.
- *Deviation* is the algebraic difference between a size and the corresponding basic size.
- *Upper deviation* is the algebraic difference between the maximum limit and the corresponding basic size.
- *Lower deviation* is the algebraic difference between the minimum limit and the corresponding basic size.
- *Fundamental deviation* is either the upper or the lower deviation, depending on which is closer to the basic size.
- *Tolerance* is the difference between the maximum and minimum size limits of a part.
- *International tolerance grade* numbers (IT) designate groups of tolerances such that the tolerances for a particular IT number have the same relative level of accuracy but vary depending on the basic size.
- *Hole basis* represents a system of fits corresponding to a basic hole size. The fundamental deviation is H.
- AncoraSIR.com shaft-basis system is not included here. • *Shaft basis* represents a system of fits corresponding to a basic shaft size. The fundamental deviation is h. The

Limits and Fits

capital letters always refer to the hole; lowercase letters are used for the shaft

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To establish a preferred fit

How the letters are combined with the tolerance grades

Fits

Sourc Limit $BA2$

- The fundamental deviations for shafts are given in Tables A–11 and A–13.
- For letter codes c, d, f, g, and h,
	- Upper deviation 5 fundamental deviation
	- Lower deviation 5 upper deviation 2 tolerance grade
- For letter codes k, n, p, s, and u, the deviations for shafts are
	- Lower deviation 5 fundamental deviation
	- Upper deviation 5 lower deviation 1 tolerance grade
- The lower deviation H (for holes) is zero.
	- For these, the upper deviation equals the tolerance grade.

Table A-11

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Table A-12

Fundamental Deviations for Shafts-Metric Series

(Size Ranges Are for Over the Lower Limit and Including the Upper Limit. All Values Are in Millimeters) Source: Preferred Metric Limits and Fits, ANSI B4.2-1978. See also BSI 4500.

Example: Find the shaft and hole dimensions for a loose running fit with a 34-mm basic size.

Looking up the tables

From Table 7–9,

- From Table A–11 is 0.160 mm.
- The symbol 34H $\Delta D = \Delta d = 0.160$ n
- Using Eq. $(7-36)$

Basic

180-250 250-315 315-400

 $D_{\text{max}} = D + \Delta L$

Table A-11

Example: Find the shaft and hole dimensions for a loose running fit with a 34-mm basic size.

Looking up the tables

- The shaft is designated as a 34c11 shaft.
- From Table A–12, the fundamental deviation is δ_F = - 0.120 mm.
- Using Eq. $(7-37)$, we get for the shaft dimensions

$$
d_{\text{max}} = d + \delta_F = 34 + (-0.120) = 33.880 \text{ mm}
$$

$$
d_{\min} = d + \delta_F - \Delta d = 34 + (-0.120) - 0.160 = 33.720
$$
mm

Table A-12

Fundamental Deviations for Shafts—Metric Series

(Size Ranges Are for Over the Lower Limit and *Including* the Upper Limit. All Values Are in Millimeters) Source: Preferred Metric Limits and Fits, ANSI B4.2-1978. See also BSI 4500.

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Next class

- Lab for Group 1: Design Consulta
- Friday 0800-1000, Nov 01
- Room 412, 5 Wisdom Valley
- **Discussion for Group 2: Design Consultation**
- [•](mailto:11930324@mail.sustech.edu.cn) Friday 0800-1000, Nov 01
- Room 202, 1 Lychee Park

Thank you!

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