ME303 Introduction to Mechanical Design

Lecture 07
Gears in General

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Agenda

Week 06, Wednesday, Oct 16, 2019

- Key Concepts
  - Types of Gears | Nomenclature | Fundamentals
  - Conjugate Action | Involute Properties | Contact Ratio
- The Forming of Gear Teeth
- Common Gears
  - Straight Bevel Gears | Parallel Helical Gears | Worm Gears
- Gear Trains
- Force Analysis
  - Spur Gearing | Bevel Gearing | Helical Gearing | Worm Gearing

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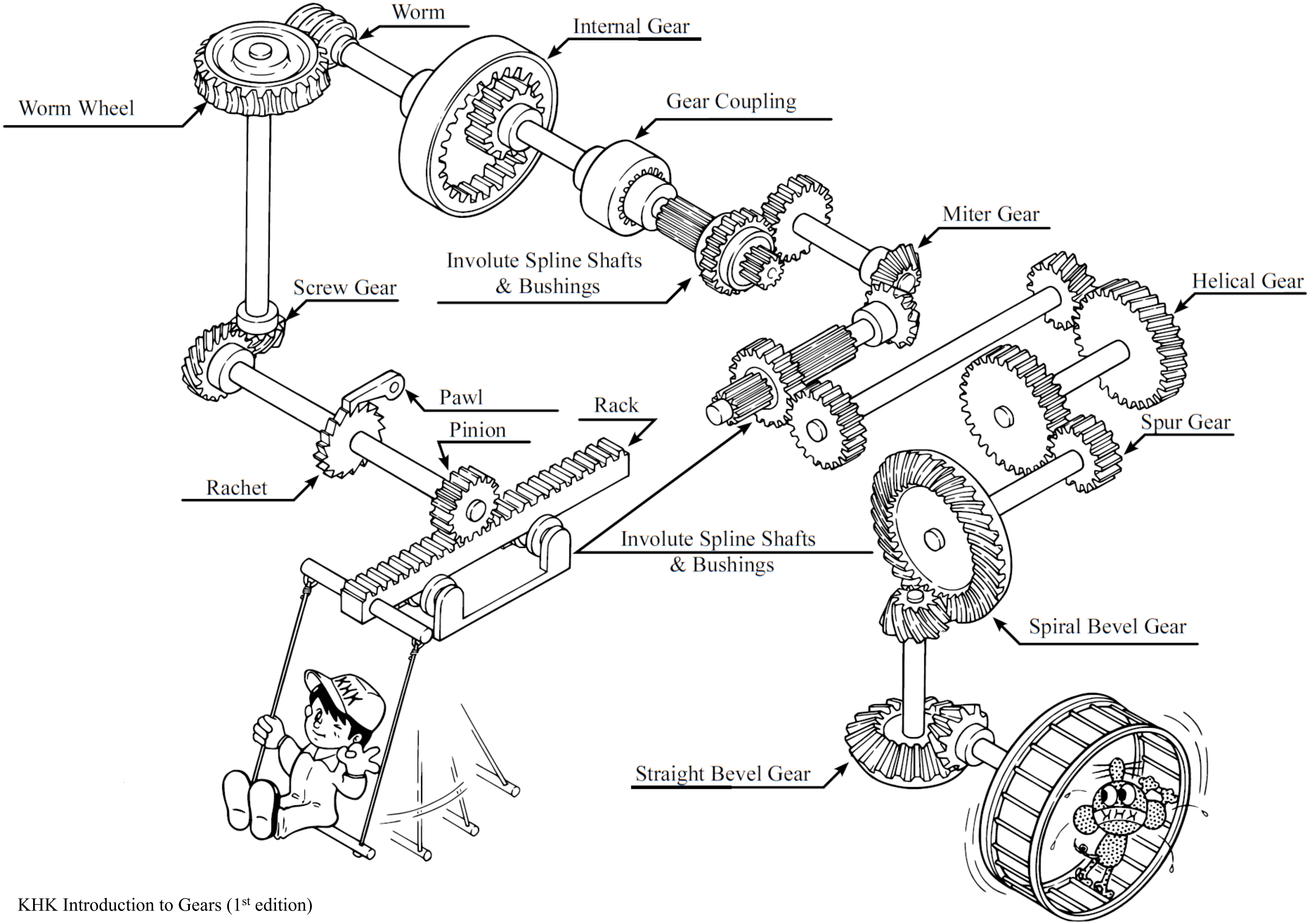
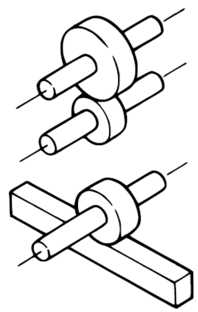


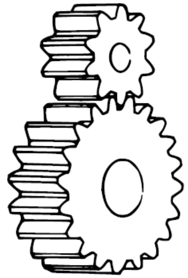
KHK Introduction to Gears (1st edition)

Types of Gears

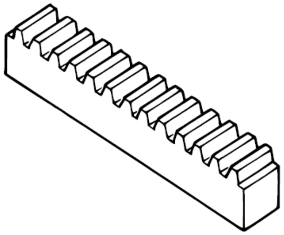
Parallel Axes



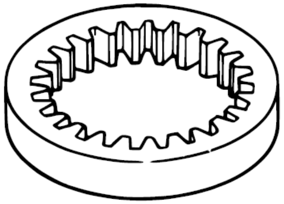
Spur Gear



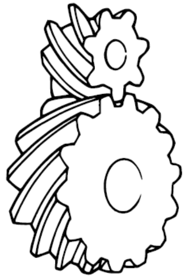
Rack



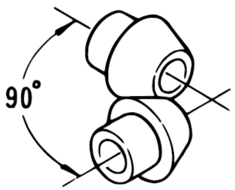
Internal Gear



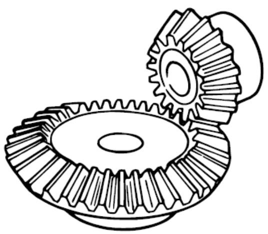
Helical Gear



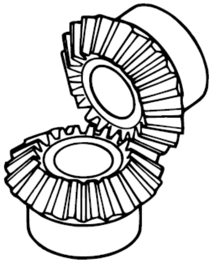
Intersecting Axes



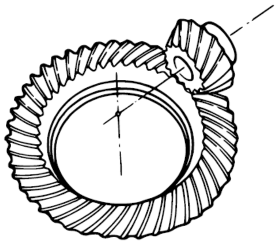
Bevel Gear



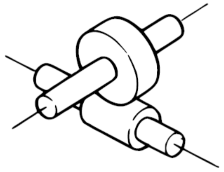
Miter Gear



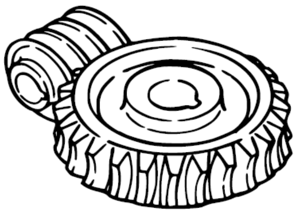
Spiral Bevel Gear



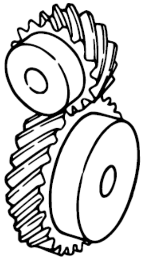
Nonparallel,
Nonintersecting
Axes



Worm & Worm Wheel



Screw Gear



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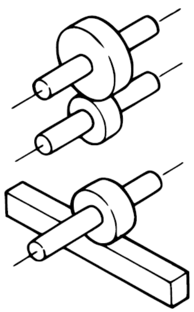
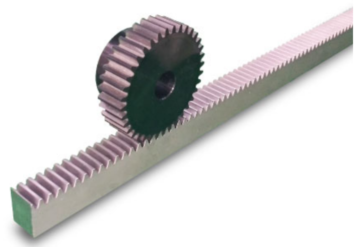
Types of Gears

Parallel Axes



Spur Gear

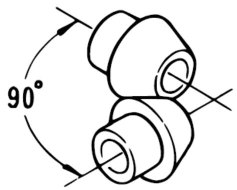
Rack



Internal Gear

Helical Gear

Intersecting Axes

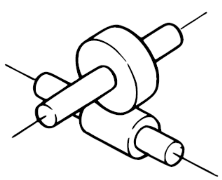


Bevel Gear

Miter Gear

Spiral Bevel Gear

Nonparallel,
Nonintersecting
Axes



Worm & Worm Wheel

Screw Gear

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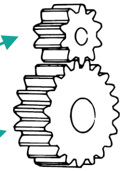
Nomenclature

Using Spur Gear as an Example

The pitch circles of a pair of mating gears are tangent to each other.

- A **pinion** is the smaller gear
- The larger is often called the **gear**



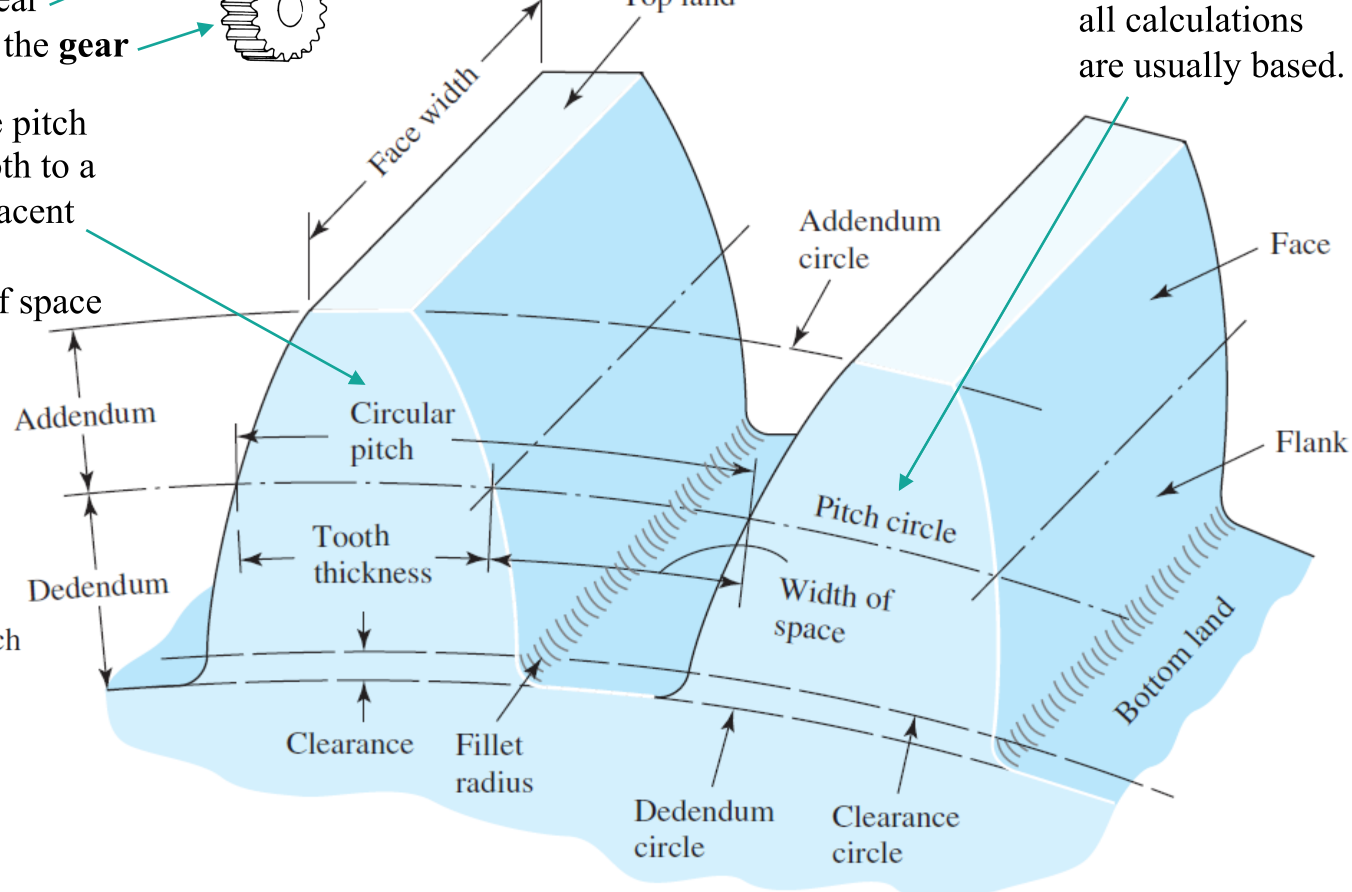
The distance, measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth

- Tooth thickness + width of space

A theoretical circle upon which all calculations are usually based.

$$P = \frac{N}{d} \qquad p = \frac{\pi d}{N} = \pi m$$

$$m = \frac{d}{N} \qquad pP = \pi$$



P = diametral pitch, teeth per inch

N = number of teeth

d = pitch diameter, in or mm

m = module, mm

p = circular pitch, in or mm

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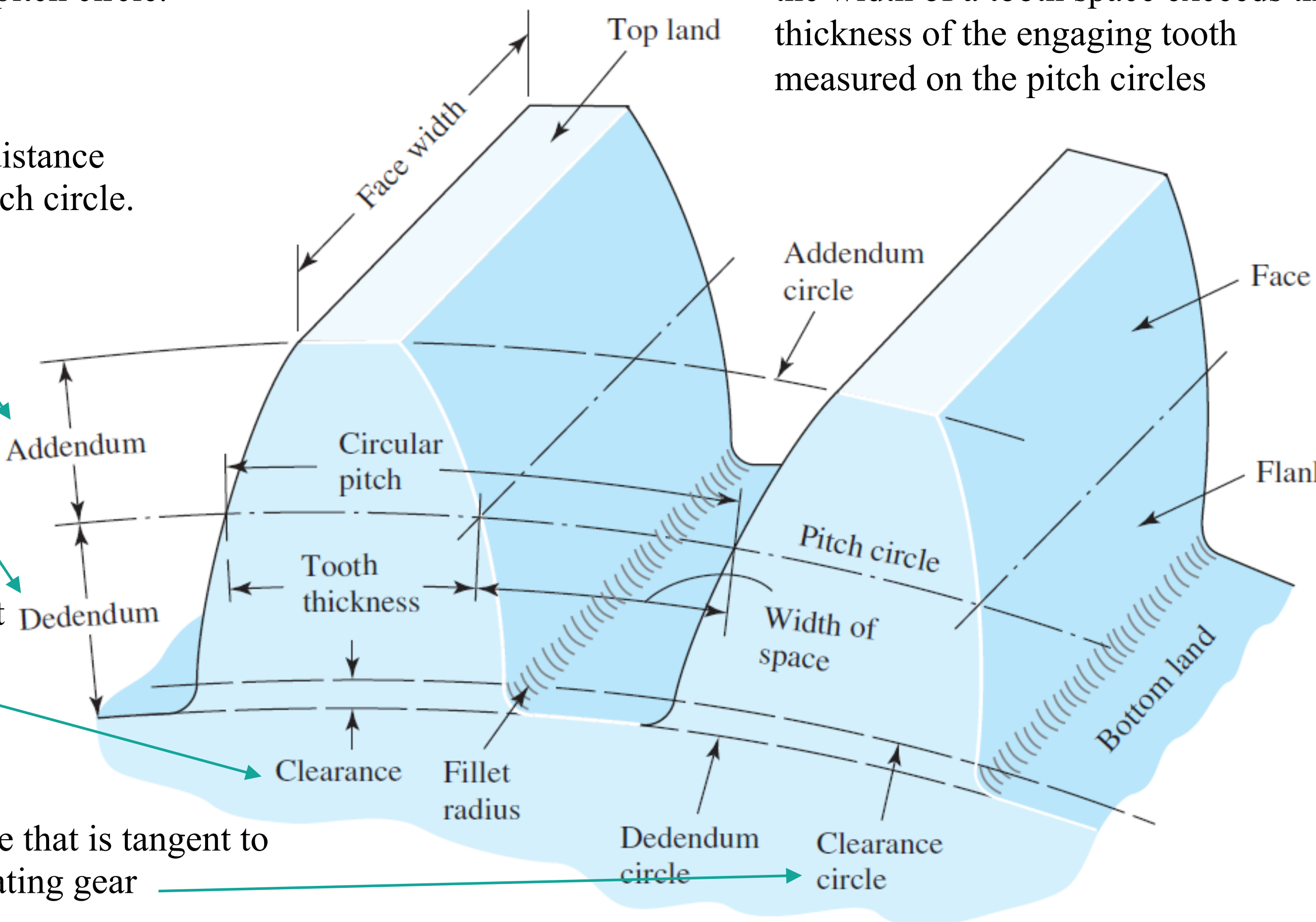
Nomenclature

Parametric Definition of a Gear

The *addendum a* is the radial distance between the *top land* and the pitch circle.

The *dedendum b* is the radial distance from the *bottom land* to the pitch circle.

The *backlash* is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circles



The *clearance c* is the amount by which the dedendum in a given gear exceeds the addendum of its mating gear.

The *clearance circle* is a circle that is tangent to the addendum circle of the mating gear

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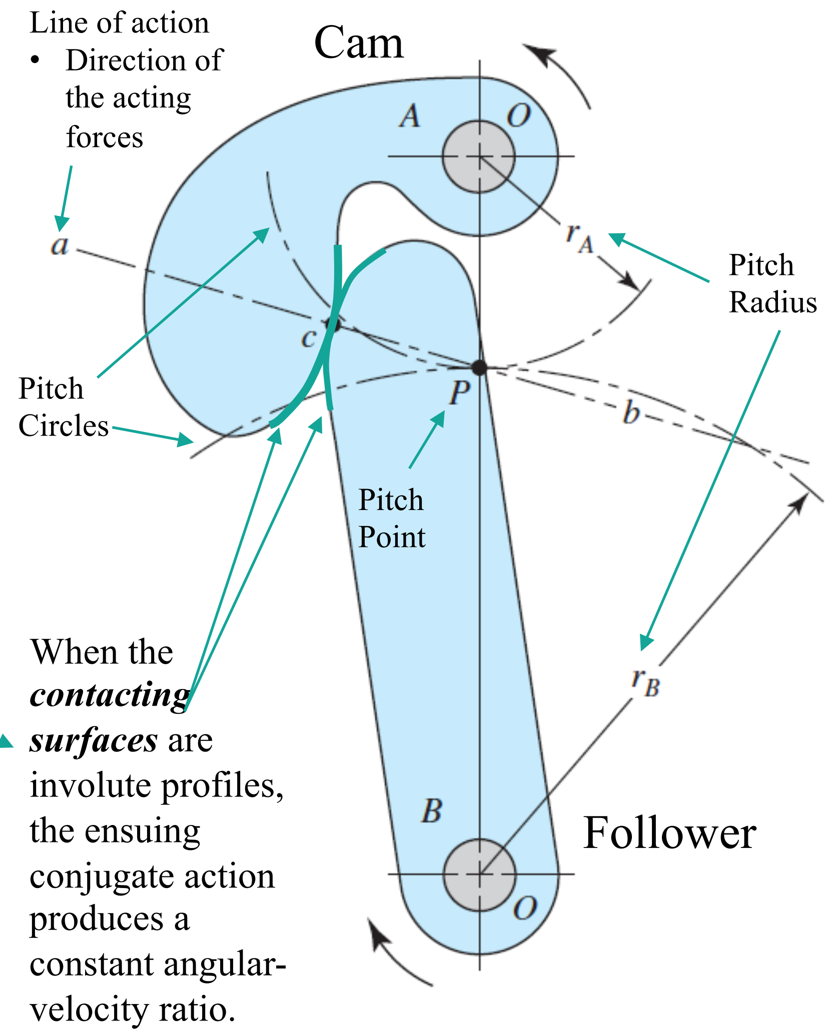
Conjugate Action

Assumes the teeth to be perfectly formed, perfectly smooth, and absolutely rigid.

- Mating Gears vs. Cams
- Conjugate Action
  - When the tooth profiles, or cams, are designed so as to produce ***a constant angular-velocity ratio*** during meshing.
- Involute Profile
  - In universal use for gear teeth and with few exceptions.
- Features
  - Conjugate action is independent of changes in center distance
  - Can be manufactured at low cost since the tooth profile is relatively simple
  - A typical tooth profile used almost exclusively for gears



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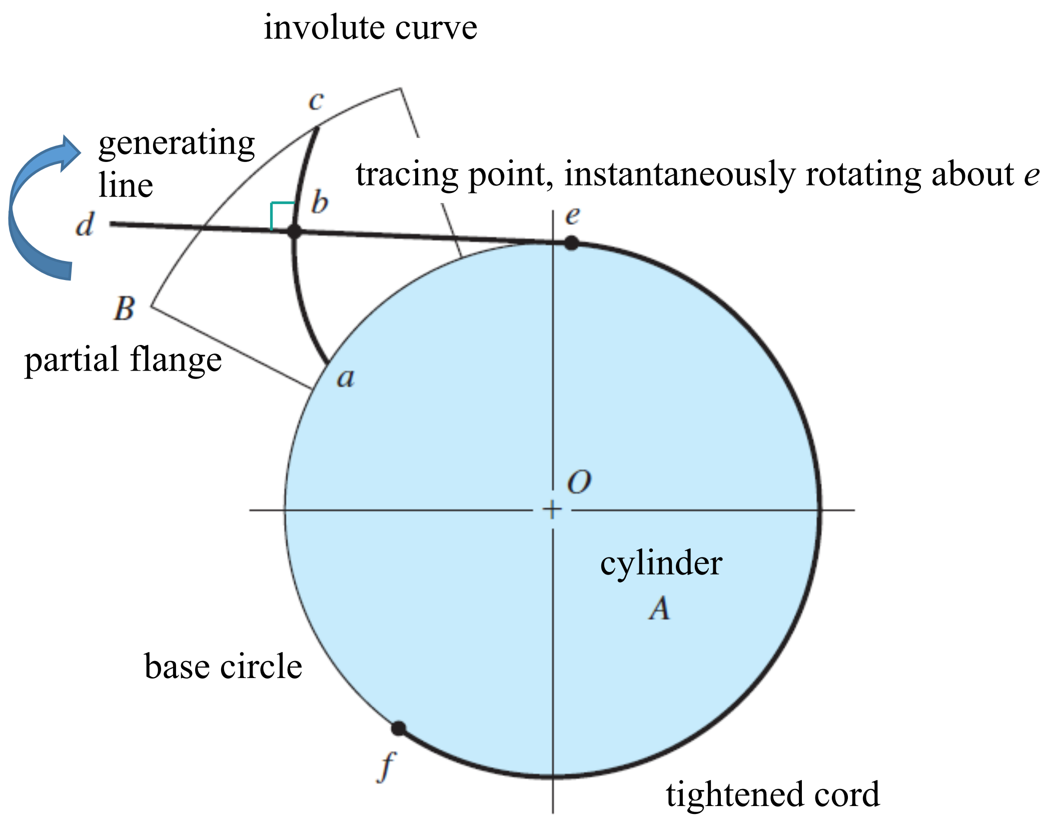
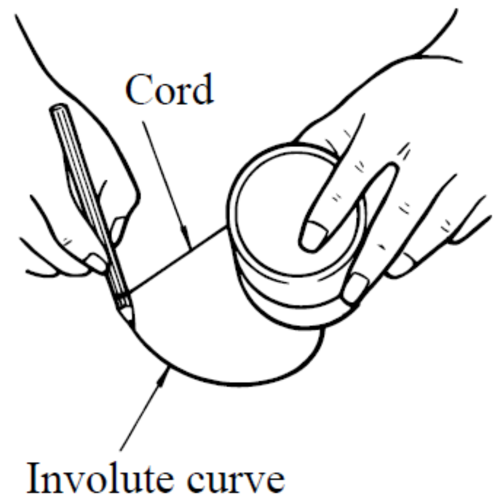
Involute Properties

Pathway to Design a Gear Curve



Generation of an Involute



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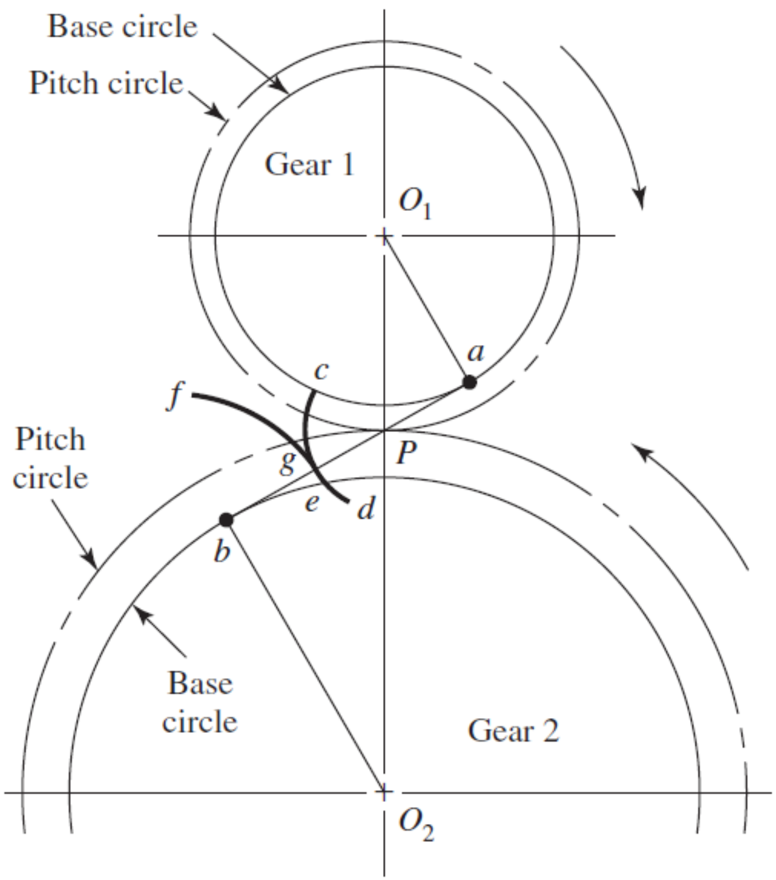
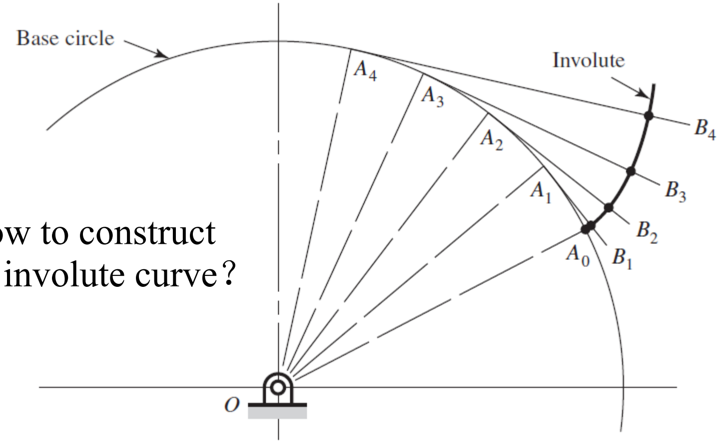
Fundamentals: Involute Curve

Basic Geometry



Involute Action



How to construct an involute curve?

1. Divide the base circle into a number of equal parts
2. Construct radial lines OA_0, OA_1, OA_2, etc.
3. Beginning at A1, construct perpendiculars A_1B_1, A_2B_2, A_3B_3, etc.
4. Along A_1B_1 lay off the distance A_1A_0,
5. Along A_2B_2 lay off **twice** the distance A_1A_0, etc.,
6. Producing points through which the involute curve can be constructed.

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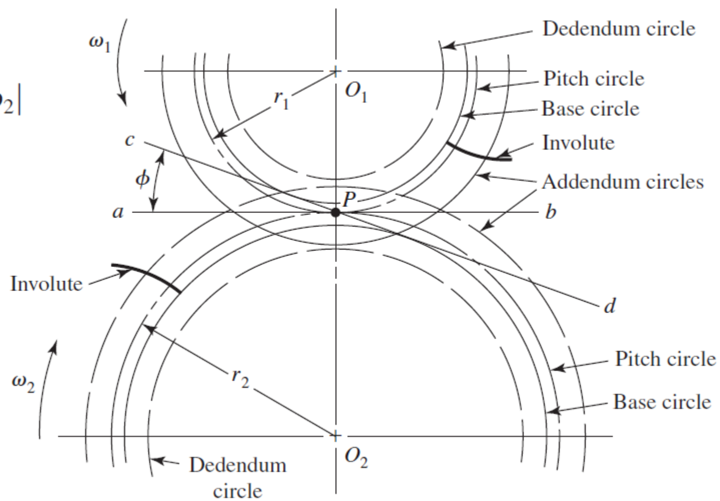
Fundamentals: Gear Profile

To draw the teeth on a pair of meshing gears?

Pitch circles *roll without slipping* and are the basis of the various dimensions found in gearing.

$$V = |r_1\omega_1| = |r_2\omega_2|$$

$$\left|\frac{\omega_1}{\omega_2}\right| = \frac{r_2}{r_1}$$



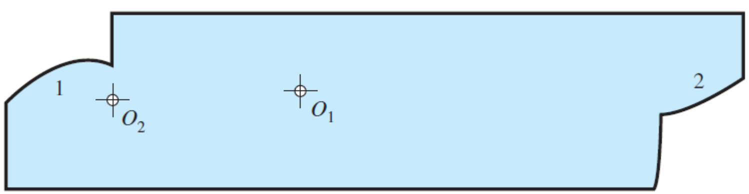
1. Locate gear centers O_1 and O_2;
2. Construct ***pitch circles*** of radii r_1 and r_2;
3. Identify tangent at P, the ***pitch point***;
4. Draw line ab, the common tangent through P;
5. Draw line cd through P at angle ϕ to ab;

Pressure Line /Generating Line/Line of Action

Pressure Angle (20˚/25˚)

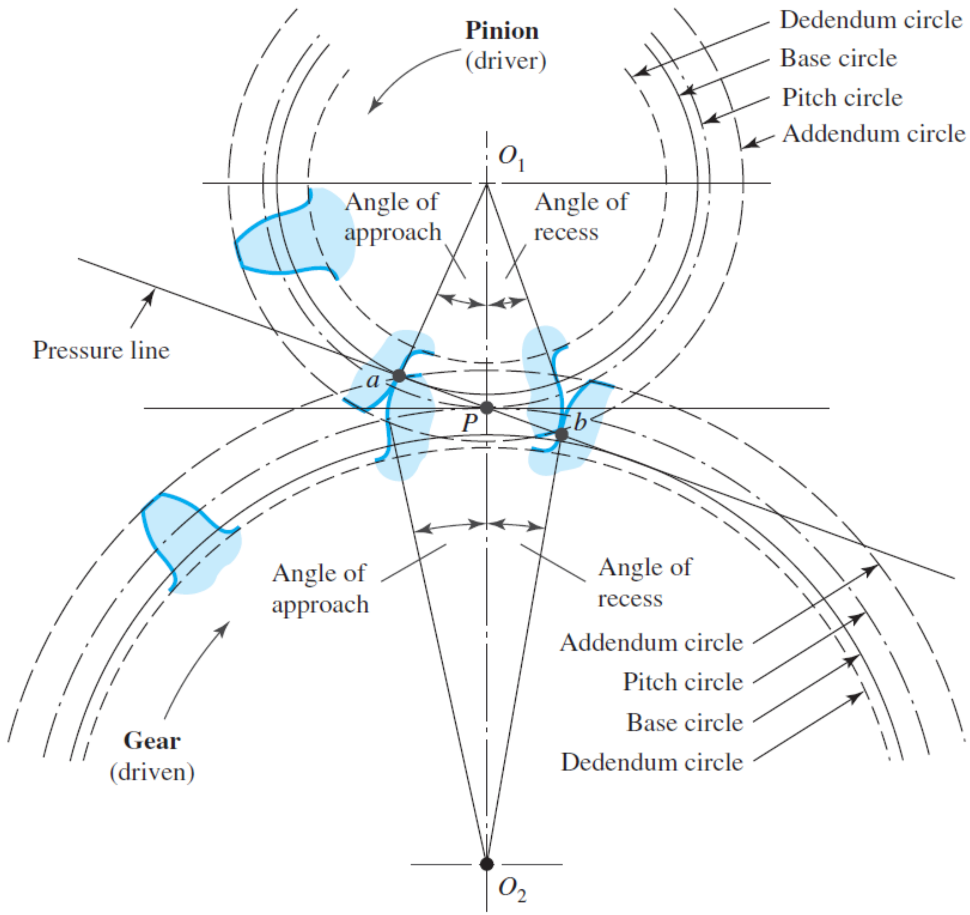
$$r_b = r\cos\phi$$

6. On each gear draw a ***base circle*** tangent to the pressure line, determined by ϕ;
7. Now generate an ***involute*** on each base circle as previously described;
8. Use **1/*P*** for the addendum distance and **1.25/*P*** for the dedendum distance to draw the ***addendum*** and ***dedendum*** circles on the pinion and on the gear.
9. Determine circular pitch using $p = \pi/P$;
10. Determine tooth thickness using $t = p/2$;
11. Prepare a template to draw the profile;
12. Finish the drawing by adding the fillets between the ***clearance circle*** and the dedendum circle.

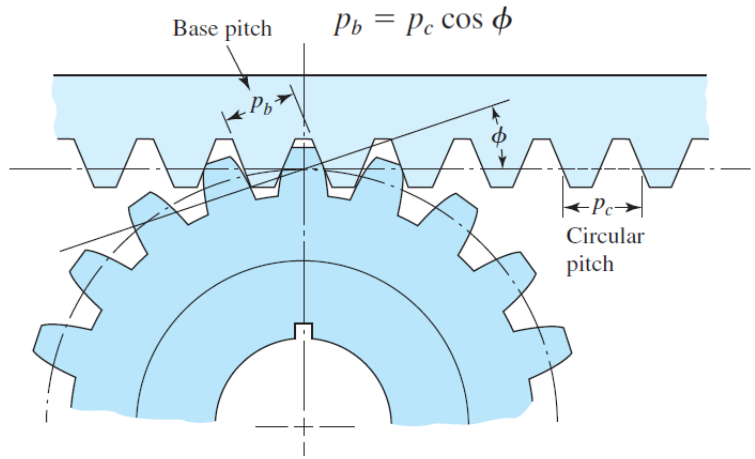


Tooth Action

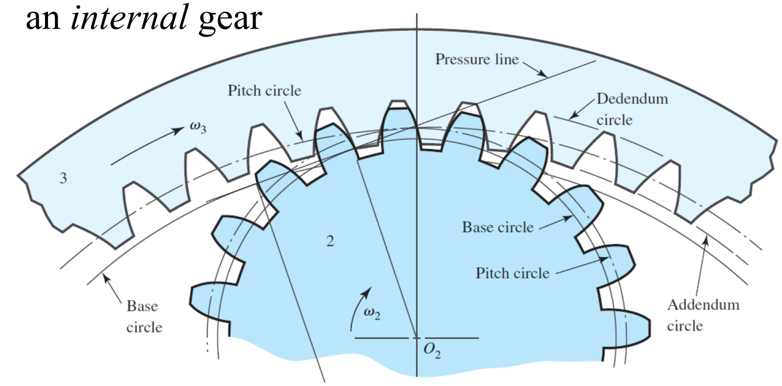
The motion of one tooth relative to the mating tooth is *primarily* a rolling motion; In fact, when contact occurs at the *pitch point*, the motion is *pure rolling*.



We may imagine a *rack* as a spur gear having an infinitely large pitch diameter.



an *internal* gear



Example on Gear Fundamentals

A gearset consists of a 16-tooth pinion driving a 40-tooth gear. The diametral pitch is 2, and the addendum and dedendum are $1/P$ and $1.25/P$, respectively. The gears are cut using a pressure angle of 20°.
(*a*) Compute the circular pitch, the center distance, and the radii of the base circles.

(*a*)
$$p = \frac{\pi}{P} = \frac{\pi}{2} = 1.571 \text{ in}$$

The pitch diameters of the pinion and gear are, respectively,

$$d_P = \frac{N_P}{P} = \frac{16}{2} = 8 \text{ in} \qquad d_G = \frac{N_G}{P} = \frac{40}{2} = 20 \text{ in}$$

Therefore the center distance is

$$\frac{d_P + d_G}{2} = \frac{8 + 20}{2} = 14 \text{ in}$$

Since the teeth were cut on the 20° pressure angle, the base-circle radii are found to be, using $r_b = r \cos \phi$,

$$r_b(\text{pinion}) = \frac{8}{2} \cos 20^\circ = 3.759 \text{ in}$$

$$r_b(\text{gear}) = \frac{20}{2} \cos 20^\circ = 9.397 \text{ in}$$

Example on Gear Fundamentals

A gearset consists of a 16-tooth pinion driving a 40-tooth gear. The diametral pitch is 2, and the addendum and dedendum are $1/P$ and $1.25/P$, respectively. The gears are cut using a pressure angle of 20°.

(*b*) In mounting these gears, the center distance was incorrectly made $\frac{1}{4}$ in larger. Compute the new values of the pressure angle and the pitch-circle diameters.

(*b*) Designating d'_P and d'_G as the new pitch-circle diameters, the $\frac{1}{4}$-in increase in the center distance requires that

$$\frac{d'_P + d'_G}{2} = 14.250 \qquad (1)$$

Also, the velocity ratio does not change, and hence

$$\frac{d'_P}{d'_G} = \frac{16}{40} \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously yields

$$d'_P = 8.143 \text{ in} \qquad d'_G = 20.357 \text{ in}$$

Since $r_b = r \cos \phi$, using either the pinion or gear, the new pressure angle is

$$\phi' = \cos^{-1} \frac{r_b(\text{pinion})}{d'_P/2} = \cos^{-1} \frac{3.759}{8.143/2} = 22.59°$$

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Contact Ratio

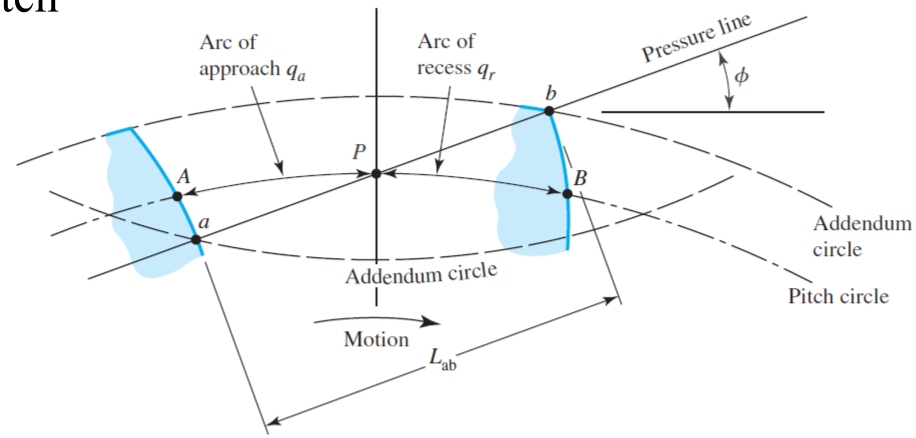
The average number of pairs of teeth in contact

- Contact Ratio

$$m_c = \frac{q_t}{p}$$

Arc of Action $q_t = q_a + q_r$

Circular pitch



- Gears should not generally be designed having contact ratios less than about 1.20,
    - Inaccuracies in mounting might reduce the contact ratio even more,
    - Increasing the possibility of impact between the teeth
    - As well as an increase in the noise level.

Tooth contact *begins* and *ends* at the intersections of the two addendum circles with the pressure line, i.e. *A* and *B*.

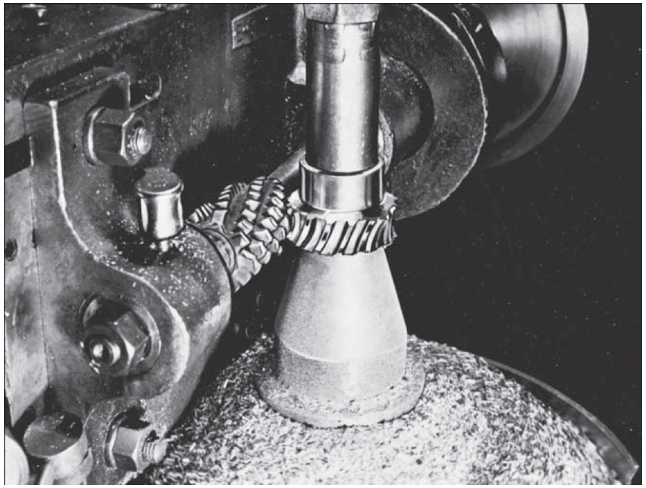
- An easier way to obtain the contact ratio is to measure the line of action *ab* instead of the arc distance *AB*.

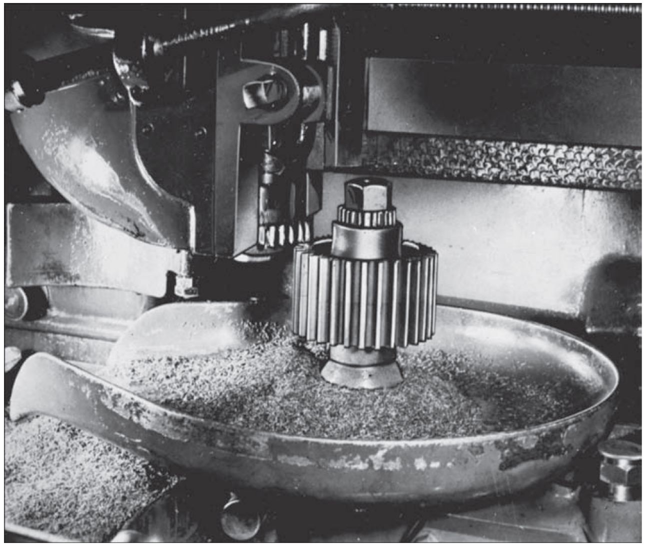
$$m_c = \frac{L_{ab}}{p \cos \phi}$$

The Forming of Gear Teeth

Milling => Shaping => Hobbing => Finishing

- Metal-based Gears
  - Common Machining Methods:
    - Milling, Shaping, or Hobbing
  - Common Finish Methods:
    - Shaving, Burnishing, Grinding, or Lapping
  - Other ways of forming the teeth of gears
    - sand casting, shell molding, investment casting, permanent-mold casting, die casting, centrifugal casting, powder-metallurgy process, extrusion, cold forming
- Thermoplastics-based Gears
  - Nylon, polycarbonate, acetal are quite popular and are easily manufactured by injection molding.
  - Low to moderate precision, low in cost for high production quantities, and capable of light loads, and can run without lubrication.





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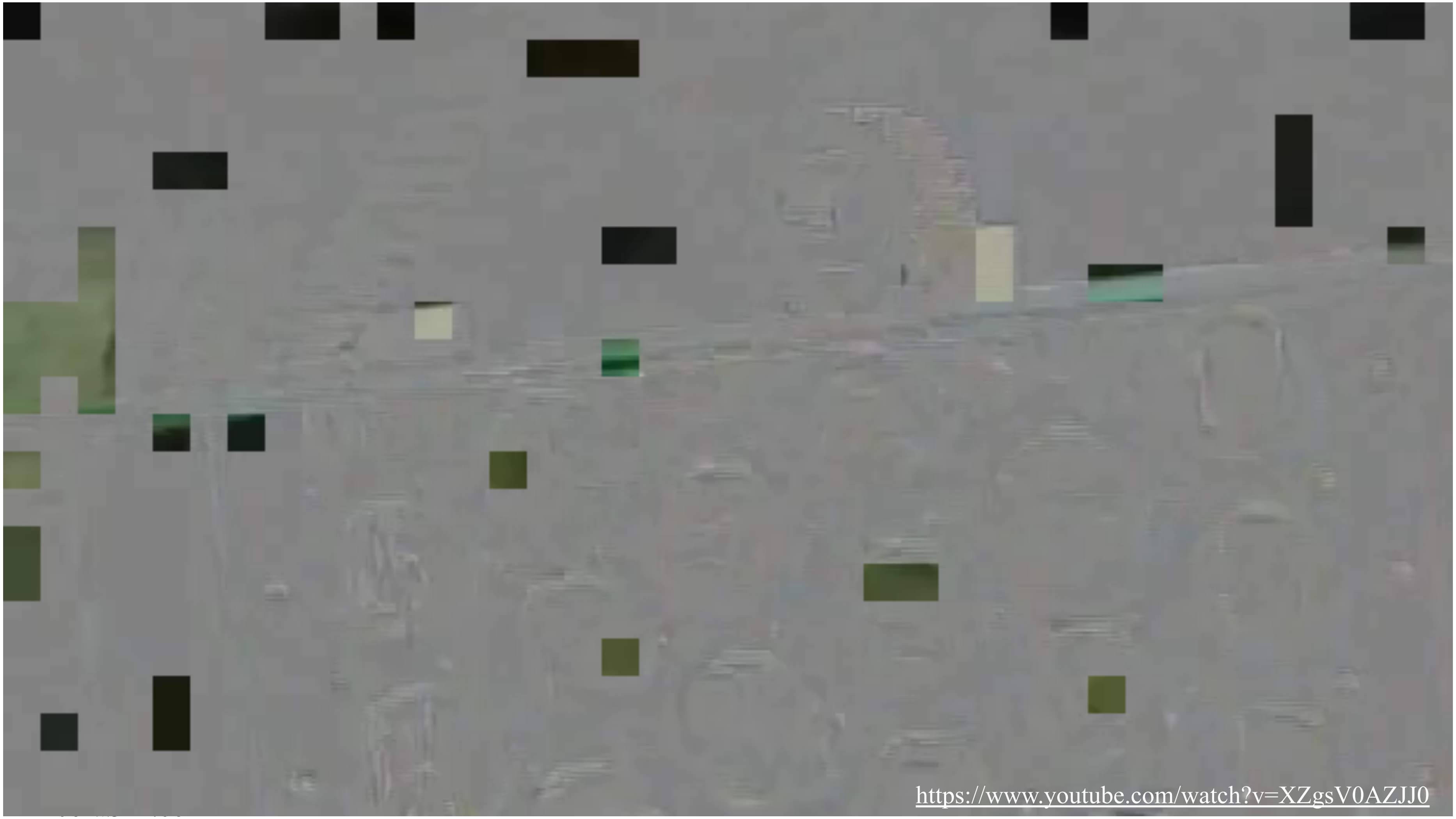
The Forming of Gear Teeth

Milling => Shaping => Hobbing => Finishing



https://www.youtube.com/watch?v=XZgsV0AZJJ0

The Making of the Spur Gears

A Typical Process of Making SS Spur Gears

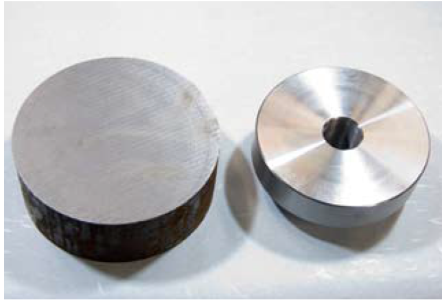
Raw Materials

Raw materials bought from material makers are kept in stock. The materials are six meters in length.

Sawing

The materials are cut to size.

Shaping



With a lathe, a cut workpiece is shaped into gear blank.

Tooth-Cutting

Tooth-cutting has been done with a gear hobbing machine. The cutting usually leaves burrs on the teeth.

Deburring

Rough spots on the teeth have been smoothed with a deburring machine.

Black Oxide Finish

The black oxide finish is somewhat effective in preventing rust.

Packaging

To ensure delivery in good condition each and every spur gear is individually packaged.

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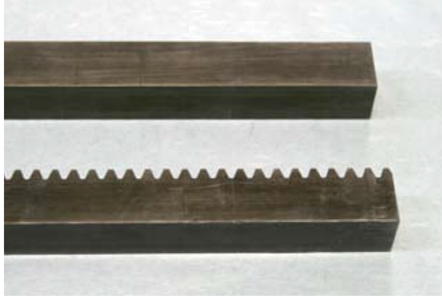
The Making of the Bevel Gears

A Typical Process of Making SM Type Bevel Gears

Raw Materials

Raw Materials bought from material makers are kept in stock.

Tooth-Cutting



Tooth-cutting has been done with a rack cutting machine. The cutting usually leaves burrs on the teeth.

Deburring

Rough spots on the teeth have been smoothed with a deburring machine.

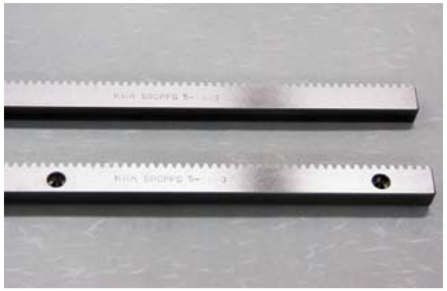
Straightening

To straighten warping, pressure is applied on racks with a hydraulic press

Machining Ends

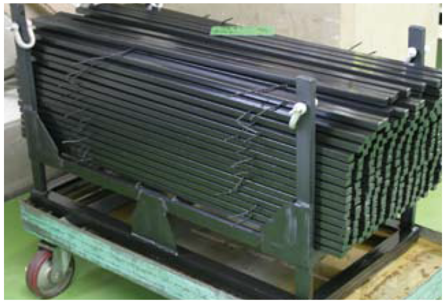
Both ends have been machined so that racks can be butted against each other to make any desired length.

Processing holes



Mounting screw holes have been drilled for easier assembly.

Black Oxide Finish



The black oxide finish is somewhat effective in preventing rust.

Packaging

To ensure delivery in good condition each and every rack is individually packaged.

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The Making of the Racks

A Typical Process of Making SRFD Type Racks

Raw aterials

Raw materials bought from material makers are kept in stock. The materials are six meters in length.

Sawing

The materials are cut to size.

Shaping

With a lathe, a cut workpiece is shaped into gear blank.

Tooth-Cutting

Tooth-cutting has been done with a Coniflex generator. The cutting usually leaves burrs on the teeth.

Deburring

Rough spots on the teeth are being smoothed with a deburring machine.

Black Oxide Finish

The black oxide finish is somewhat effective in preventing rust.

Packaging

To ensure delivery in good condition each and every bevel gear is individually packaged.

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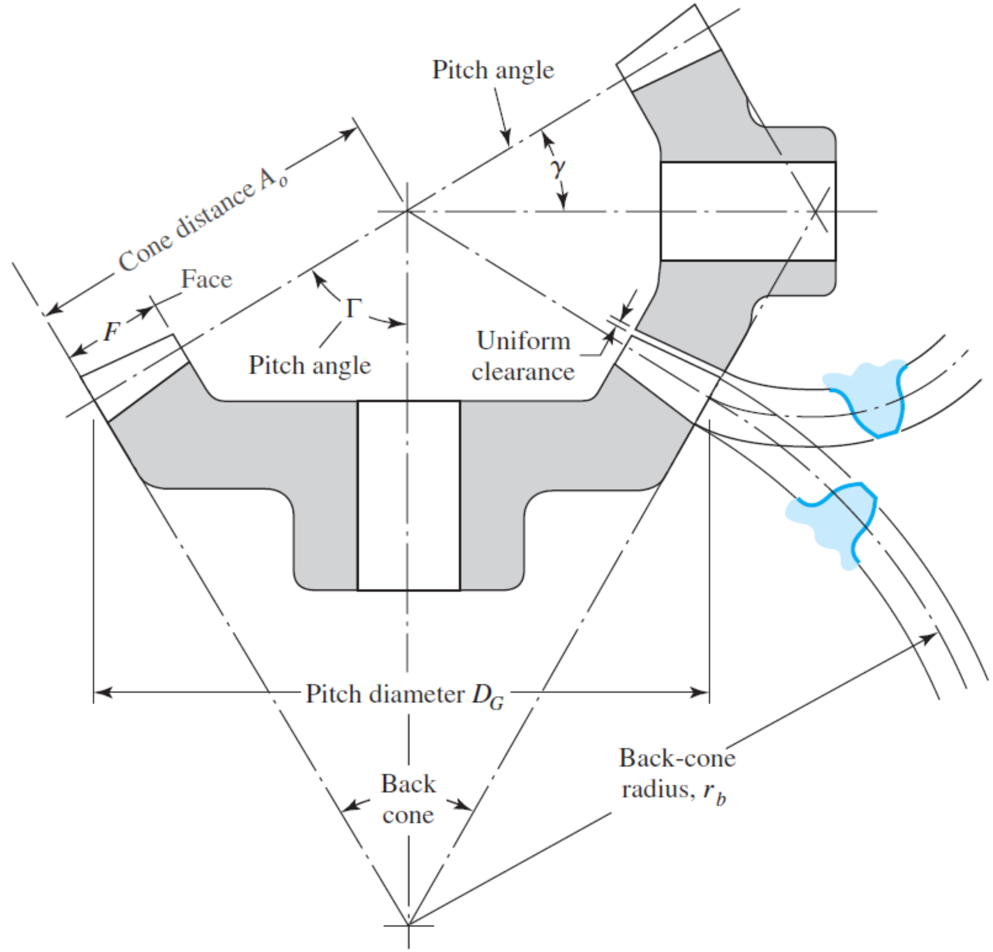
Straight Bevel Gears

When gears are used to transmit motion between intersecting shafts

- The pitch of bevel gears
  - measured at the large end of the tooth
  - both the circular pitch and the pitch diameter are calculated in the same manner as for spur gears

- Pitch Angles
  - Defined by the pitch cones meeting at the apex

$$\tan \gamma = \frac{N_P}{N_G} \qquad \tan \Gamma = \frac{N_G}{N_P}$$



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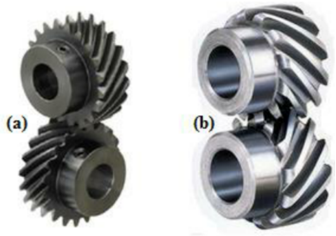
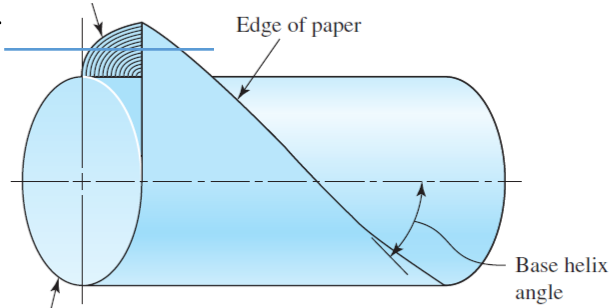
Parallel Helical Gears



transmit motion between parallel shafts

- The helix angle is the same on each gear, but one gear must have a right-hand helix and the other a left-hand helix.
- Gradual engagement of the teeth
  - The initial contact is a point that extends into a line as the teeth come into more engagement
  - Enables a smooth transfer of load from one tooth to another
  - The ability to transmit heavy loads at high speeds
    - **Contact Ratio** is of only minor importance
    - **Contact Area** becomes significant, proportional to the face width of the gear



- Loading Types: Radial & Thrust
  - Minimum thrust load as a design preference
- Double Helical Gears
  - When the thrust loads become high
  - Equivalent to two helical gears of opposite hand, mounted side by side on the same shaft
  - Develop opposite thrust reactions and thus cancel out the thrust load.

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Example on Helical Gear

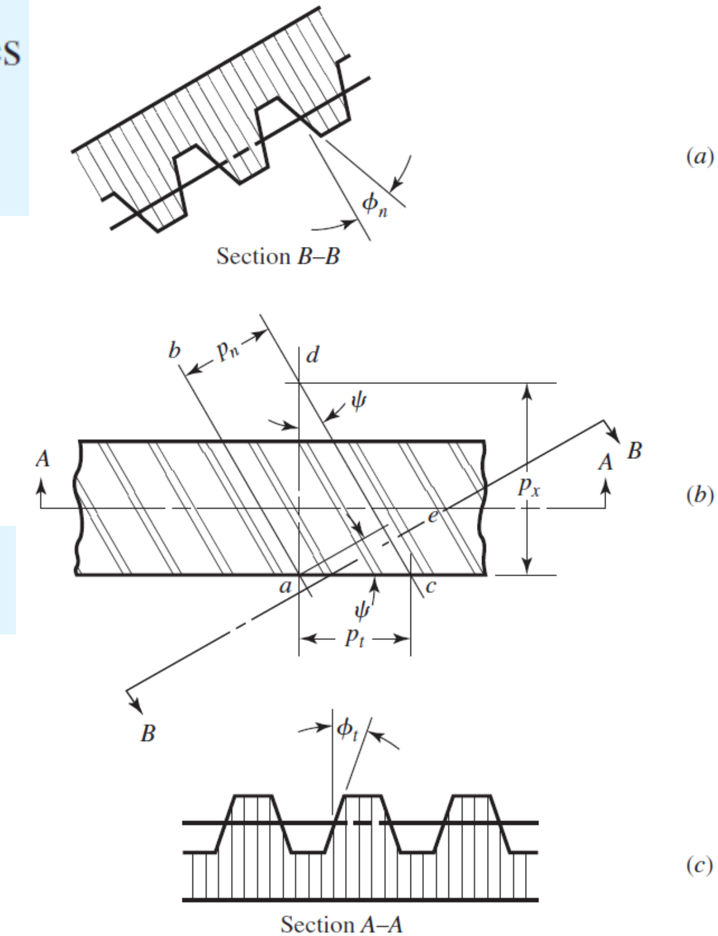
A stock helical gear has a normal pressure angle of 20°, a helix angle of 25°, and a transverse diametral pitch of 6 teeth/in, and has 18 teeth. Find:

(*a*) The pitch diameter

(*b*) The transverse, the normal, and the axial pitches

(*c*) The normal diametral pitch

(*d*) The transverse pressure angle



$$d = \frac{N}{P_t} = \frac{18}{6} = 3 \text{ in}$$

$$P_n = \frac{P_t}{\cos\psi} = \frac{6}{\cos 25°} = 6.620 \text{ teeth/in}$$

$$p_t = \frac{\pi}{P_t} = \frac{\pi}{6} = 0.5236 \text{ in}$$

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20°}{\cos 25°}\right) = 21.88°$$

$$p_n = p_t \cos\psi = 0.5236 \cos 25° = 0.4745 \text{ in}$$

$$p_x = \frac{p_t}{\tan\psi} = \frac{0.5236}{\tan 45°} = 1.123 \text{ in}$$

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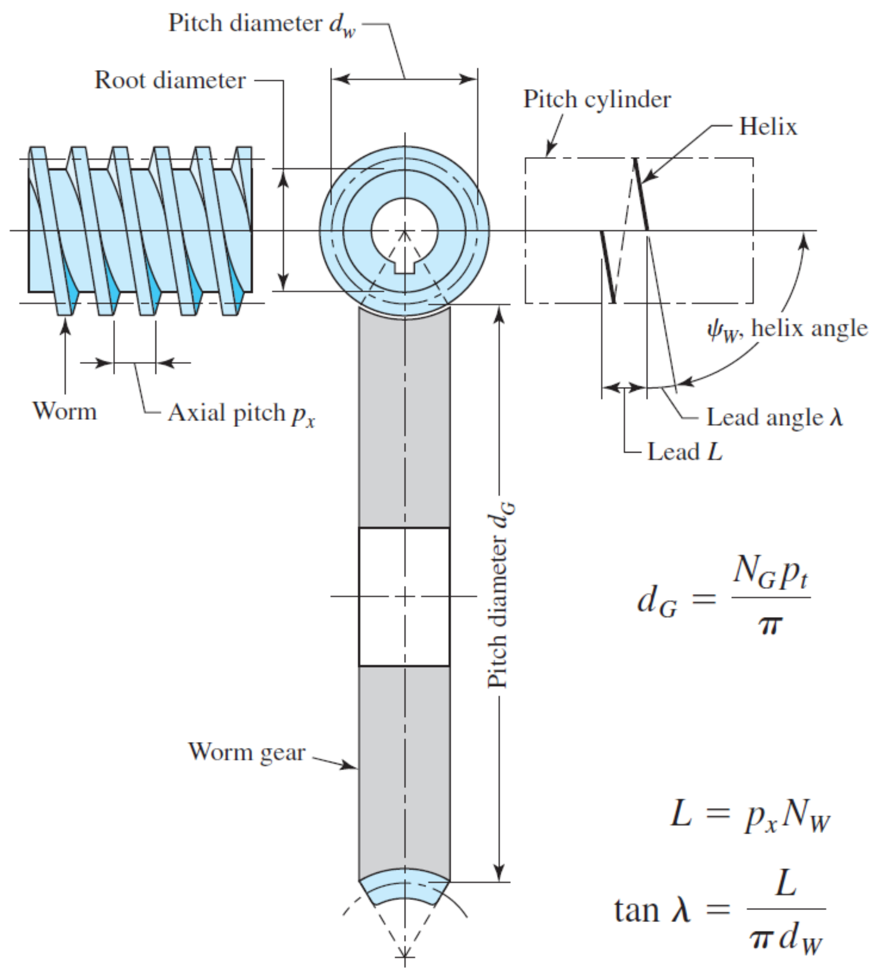
Worm Gears

The helix angle on the worm is generally quite large, and that on the gear very small.

- The worm and worm gear of a set have the same hand of helix as for crossed helical gears, but the helix angles are usually quite different.
  - The *helix angle* on the **worm** is generally quite large
    - thus *lead angle* λ on the **worm** is often used
  - The *helix angle* ψ_G on the **gear** very small
  - $\lambda = \psi_G$ for a 90˚ shaft angle.



$$d_G = \frac{N_G p_t}{\pi}$$

$$L = p_x N_W$$

$$\tan \lambda = \frac{L}{\pi d_W}$$

- Pitch of the Worm Gearsets
  - p_x: Axial Pitch of the worm
  - p_t: Transverse Circular Pitch of the mating gear
  - $p_x = p_t$ for a 90˚ shaft angle

- Generally, the pitch diameter of the worm should be selected so as to fall into the range

$$\frac{C^{0.875}}{3.0} \le d_W \le \frac{C^{0.875}}{1.7}$$

C is the center distance

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Gear Trains

- Consider a pinion 2 driving a gear 3. The speed of the driven gear is

$$n_3 = \left|\frac{N_2}{N_3}n_2\right| = \left|\frac{d_2}{d_3}n_2\right|$$

where n = revolutions or rev/min
N = number of teeth
d = pitch diameter

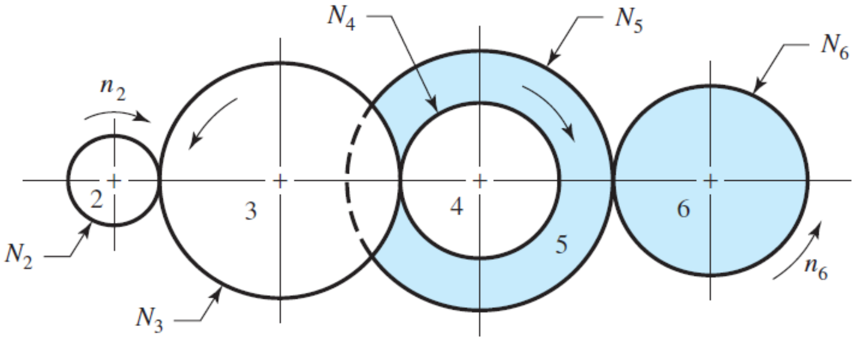
The absolute-value signs are used to permit complete freedom in choosing positive and negative directions.

- Train Value

$$e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}}$$

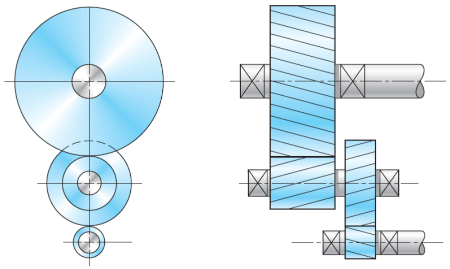
$$n_L = e n_F$$

where n_L is the speed of the last gear in the train and n_F is the speed of the first.



$$n_6 = -\frac{N_2}{N_3}\frac{N_3}{N_4}\frac{N_5}{N_6}n_2$$

- As a rough guideline,
  - A train value of up to 10 to 1 can be obtained with one pair of gears.
  - Greater ratios can be obtained in less space and with fewer dynamic problems by compounding additional pairs of gears.
  - A two-stage compound gear train can obtain a train value of up to 100 to 1.



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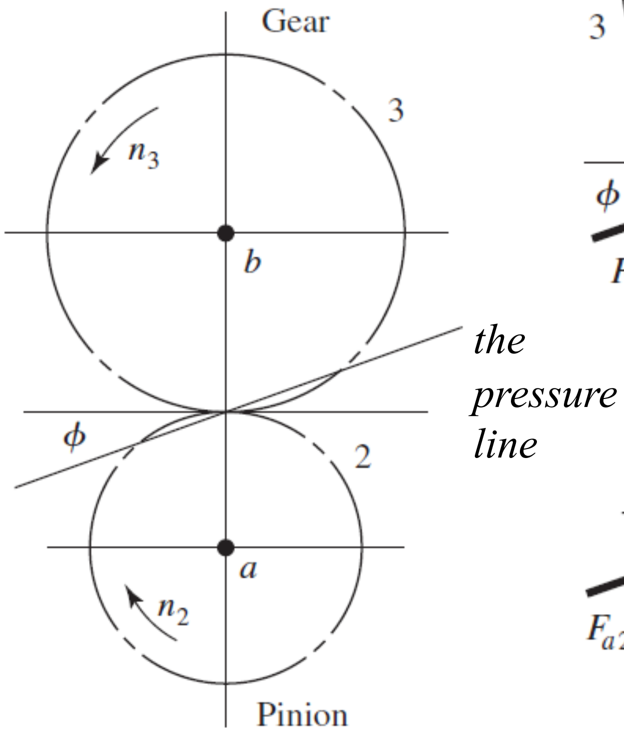
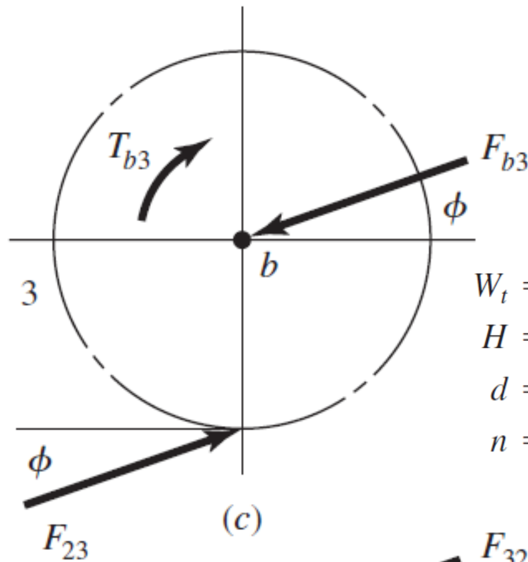
Force Analysis – Spur Gearing

A pinion mounted on shaft a rotating clockwise at n_2 rev/min and driving a gear on shaft b at n_3 rev/min.

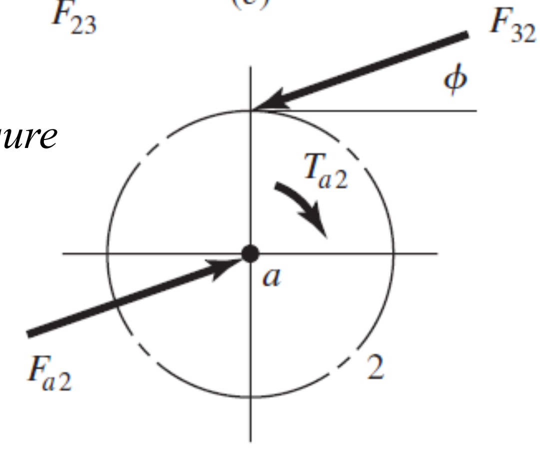
1: frame of the machine
2: input gear
3, 4, etc.: gears next
a, b, c, etc.: shafts
F_{xy}: forced excreted by x against y
r/t: radial or tangential components



(a)



(c)



(b)

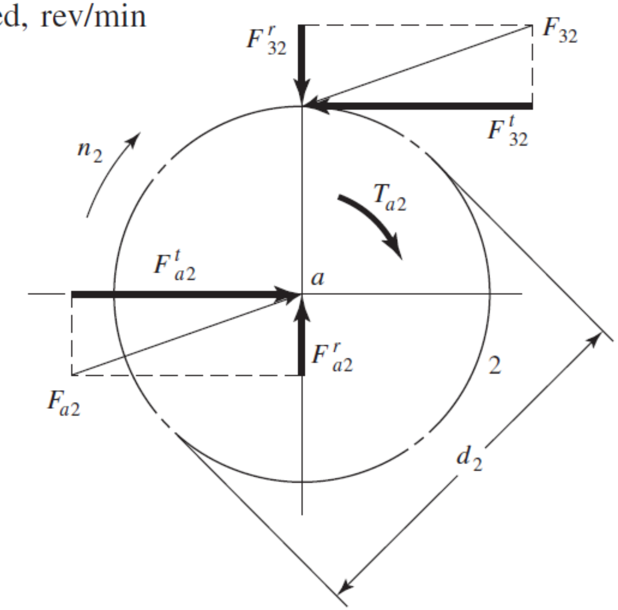
transmitted load $W_t = F^t_{32}$

applied torque $T = \frac{d}{2} W_t$

transmitted power $H = T\omega = (W_t d/2)\omega$

W_t = transmitted load, kN
H = power, kW
d = gear diameter, mm
n = speed, rev/min

$$W_t = \frac{60\,000 H}{\pi d n}$$



Example – Spur Gearing

Pinion 2 runs at 1750 rev/min and transmits 2.5 kW to idler gear 3. The teeth are cut on the 20° full-depth system and have a module of m = 2.5 mm. Analyze the forces on gear 3.

- The pitch diameters

$$d_2 = N_2 m = 20(2.5) = 50 \text{ mm}$$
$$d_3 = N_3 m = 50(2.5) = 125 \text{ mm}$$

- The transmitted load

$$W_t = \frac{60\,000H}{\pi d_2 n} = \frac{60\,000(2.5)}{\pi(50)(1750)} = 0.546 \text{ kN}$$

- The tangential force of gear 2 on gear 3

$$F^r_{23} = F^t_{23} \tan 20° = (0.546) \tan 20° = 0.199 \text{ kN}$$

$$F_{23} = \frac{F^t_{23}}{\cos 20°} = \frac{0.546}{\cos 20°} = 0.581 \text{ kN}$$

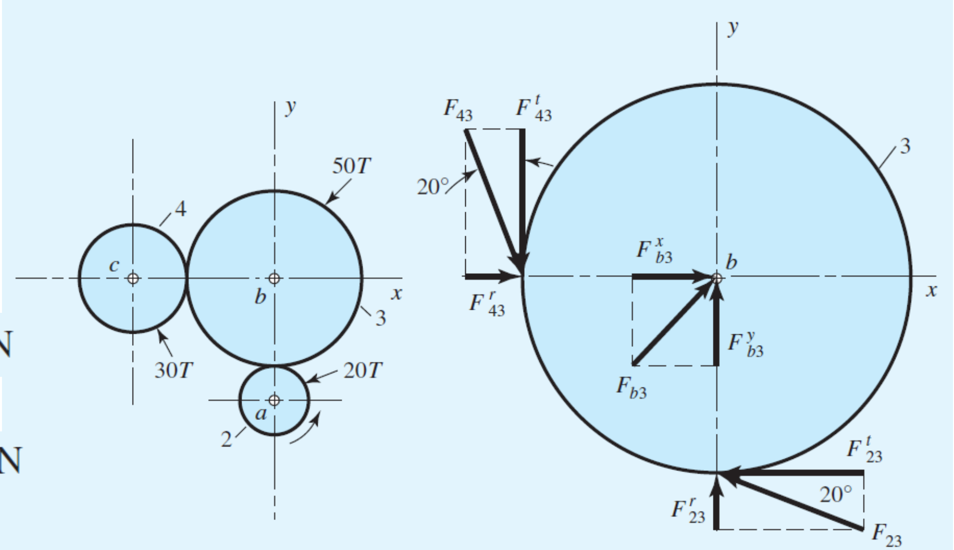
- *Idler gear transmits no power to its shaft*, so the tangential reaction of gear 4 on gear 3 is also equal to the transmitted load

$$F^t_{43} = 0.546 \text{ kN} \qquad F^r_{43} = 0.199 \text{ kN} \qquad F_{43} = 0.581 \text{ kN}$$

$$F^x_{b3} = -(F^t_{23} + F^r_{43}) = -(-0.546 + 0.199) = 0.347 \text{ kN}$$
$$F^y_{b3} = -(F^r_{23} + F^t_{43}) = -(0.199 - 0.546) = 0.347 \text{ kN}$$

The resultant shaft reaction

$$F_{b3} = \sqrt{(0.347)^2 + (0.347)^2} = 0.491 \text{ kN}$$



(a) (b)

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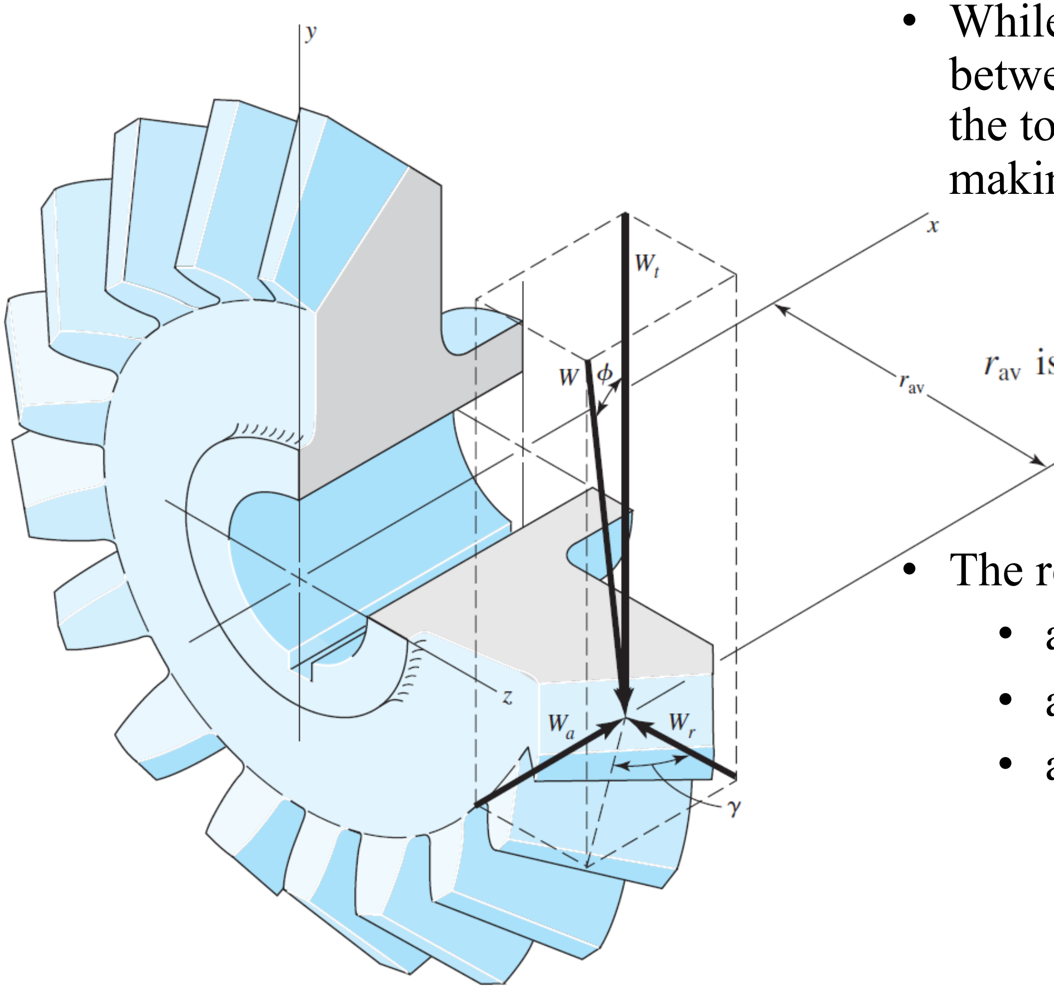
Force Analysis - Bevel Gearing

Use transmitted load that would occur if all the forces were concentrated at the midpoint of the tooth



- While the actual resultant occurs somewhere between the midpoint and the large end of the tooth, there is only a small error in making this assumption.

$$W_t = \frac{T}{r_{av}}$$

r_{av} is the pitch radius at the midpoint of the tooth

- The resultant force W has three components:
  - a tangential force,
  - a radial force, and
  - an axial force.

$$W_r = W_t \tan\phi \cos\gamma$$

$$W_a = W_t \tan\phi \sin\gamma$$

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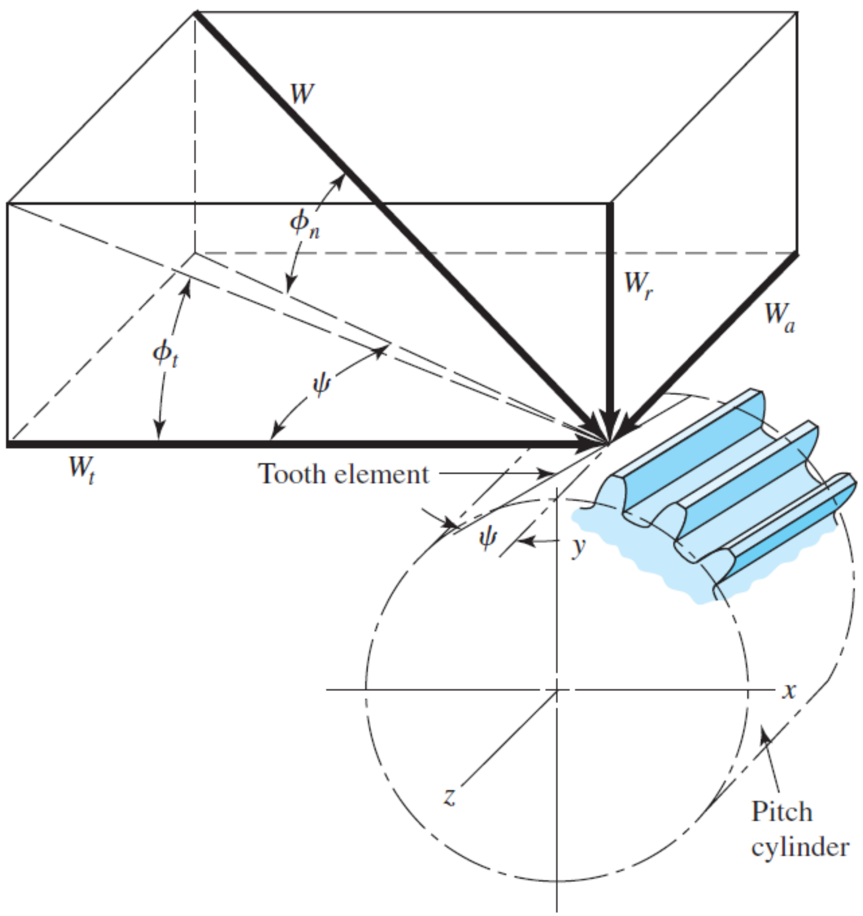
Force Analysis - Helical Gearing

The point of application of the forces is in the pitch plane and in the center of the gear face.



- The three components of the total (normal) tooth force W are

$$W_r = W \sin \phi_n$$

$$W_t = W \cos \phi_n \cos \psi$$

$$W_a = W \cos \phi_n \sin \psi$$

W = total force

W_r = radial component

W_t = tangential component, also called the transmitted load

W_a = axial component, also called the thrust load

- Usually W_t is given and the other forces are desired

$$W_r = W_t \tan \phi_t$$

$$W_a = W_t \tan \psi$$

$$W = \frac{W_t}{\cos \phi_n \cos \psi}$$

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Example – Helical Gearing

An electric motor transmits 1-hp at 1800 rev/min in the clockwise direction, as viewed from the positive x axis. Keyed to the motor shaft is an 18-tooth helical pinion having a normal pressure angle of 20˚, a helix angle of 30˚, and a normal diametral pitch of 12 teeth/in.

Make a three-dimensional sketch of the motor shaft and pinion, and show the forces acting on the pinion and the bearing reactions at A and B. The thrust should be taken out at A.

- The transverse pressure angle

$$\phi_t = \tan^{-1}\frac{\tan\phi_n}{\cos\psi} = \tan^{-1}\frac{\tan 20^\circ}{\cos 30^\circ} = 22.8^\circ$$

- The transverse pitch

$$P_t = P_n\cos\psi = 12\cos 30^\circ = 10.39 \text{ teeth/in}$$

- Therefore the pitch diameter of the pinion

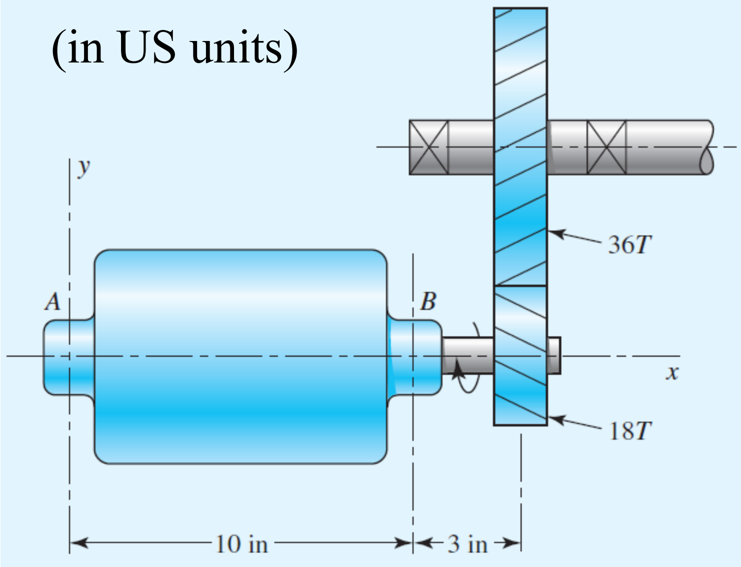
$$d_p = 18/10.39 = 1.732 \text{ in}$$

- The pitch-line velocity

$$V = \frac{\pi d n}{12} = \frac{\pi(1.732)(1800)}{12} = 816 \text{ ft/min}$$

- The transmitted load $W_t = \dfrac{33\,000H}{V} = 40.4 \text{ lbf}$

(in US units)



$$W_r = W_t\tan\phi_t = (40.4)\tan 22.8^\circ = 17.0 \text{ lbf}$$

$$W_a = W_t\tan\psi = (40.4)\tan 30^\circ = 23.3 \text{ lbf}$$

$$W = \frac{W_t}{\cos\phi_n\cos\psi} = \frac{40.4}{\cos 20^\circ\cos 30^\circ} = 49.6 \text{ lbf}$$

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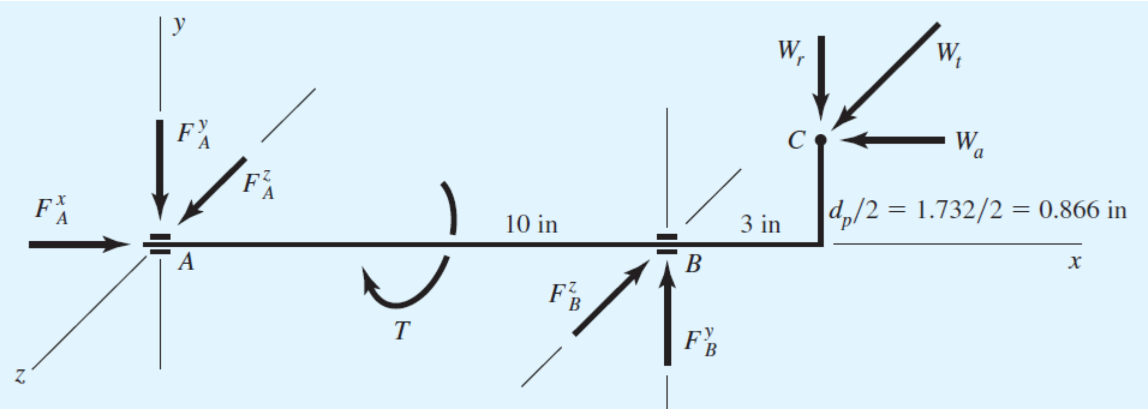
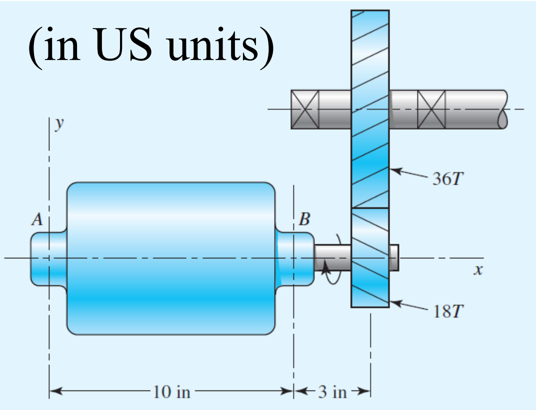
Example – Helical Gearing





- We assume bearing reactions at A and B as above: $F_A^x = W_a = 23.3$ lbf.
- Taking moments about the z axis: $-(17.0)(13) + (23.3)(0.866) + 10F_B^y = 0$ $F_B^y = 20.1$ lbf.
- Summing forces in the *y* direction: $F_A^y = 3.1$ lbf.
- Taking moments about the *y* axis: $10F_B^z - (40.4)(13) = 0$ $F_B^z = 52.5$ lbf.
- Summing forces in the *z* direction: $F_A^z = 12.1$ lbf.
- The torque: $T = W_t d_p/2 = (40.4)(1.732/2) = 35$ lbf · in

- *Can be alternatively solved by using vector method (refer to the textbook)*

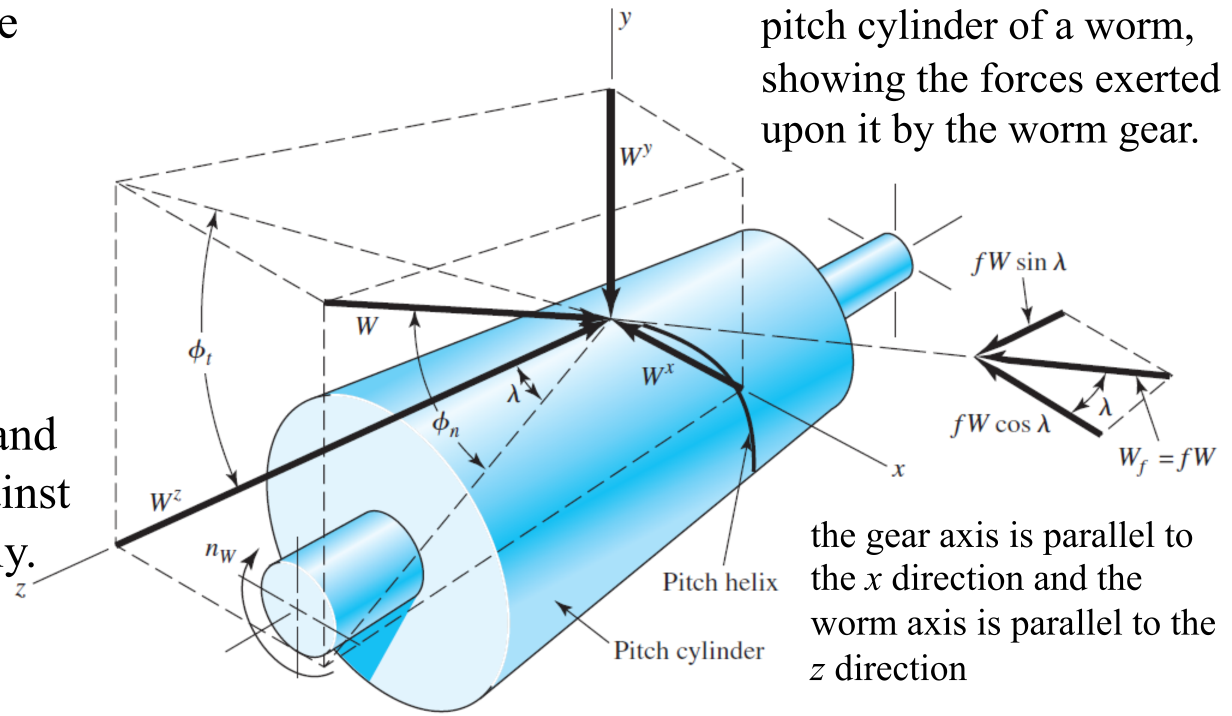
Force Analysis - Worm Gearing

The relative motion between worm and worm-gear teeth is *pure sliding*, and so we must expect that *friction* plays an important role in the performance of worm gearing.

If friction is neglected, then the only force exerted by the gear will be the force W.

$$W^x = W \cos \phi_n \sin \lambda$$
$$W^y = W \sin \phi_n$$
$$W^z = W \cos \phi_n \cos \lambda$$

pitch cylinder of a worm, showing the forces exerted upon it by the worm gear.



the gear axis is parallel to the *x* direction and the worm axis is parallel to the *z* direction

We now use the subscripts *W* and *G* to indicate forces acting against the worm and gear, respectively.

$$W_{Wt} = -W_{Ga} = W^x$$
$$W_{Wr} = -W_{Gr} = W^y$$
$$W_{Wa} = -W_{Gt} = W^z$$

The force *W* acting normal to the worm-tooth profile produces a frictional force $W_f = fW$

$$W^x = W(\cos \phi_n \sin \lambda + f \cos \lambda)$$
$$W^y = W \sin \phi_n$$
$$W^z = W(\cos \phi_n \cos \lambda - f \sin \lambda)$$

$$W_f = fW = \frac{f W_{Gt}}{f \sin \lambda - \cos \phi_n \cos \lambda}$$

Friction & Sliding of the Worm Gearing

The relative motion between worm and worm-gear teeth is *pure sliding*, and so we must expect that *friction* plays an important role in the performance of worm gearing.

- Efficiency $\eta = \dfrac{W_{Wt}\,(\text{without friction})}{W_{Wt}\,(\text{with friction})}$

$$\eta = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda}$$

- A useful relation between the two tangential forces

$$W_{Wt} = -W_{Ga} = W^x \qquad W^x = W(\cos\phi_n \sin\lambda + f\cos\lambda)$$

$$W_{Wr} = -W_{Gr} = W^y \qquad W^y = W\sin\phi_n$$

$$W_{Wa} = -W_{Gt} = W^z \qquad W^z = W(\cos\phi_n \cos\lambda - f\sin\lambda)$$

$$W_{Wt} = W_{Gt}\frac{\cos\phi_n \sin\lambda + f\cos\lambda}{f\sin\lambda - \cos\phi_n\cos\lambda}$$

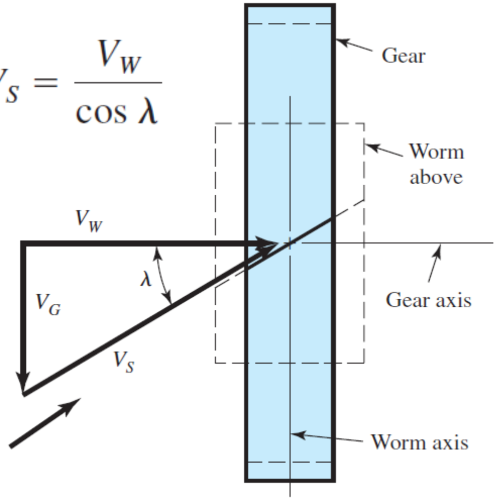
$f = 0$

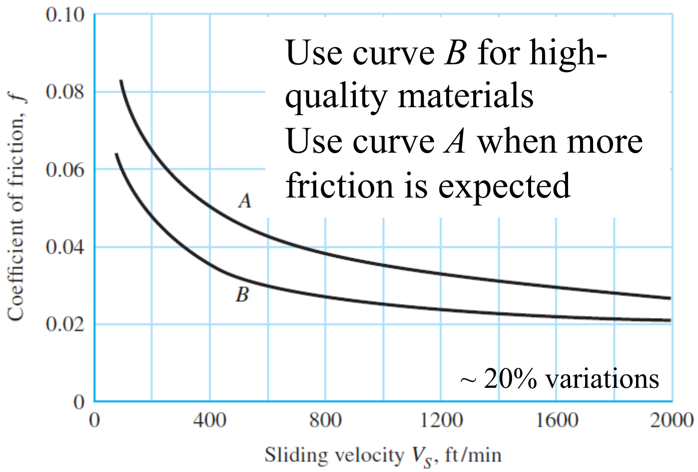
Efficiency of Worm Gearsets for $f = 0.05$

Lead Angle λ, deg	Efficiency η, %
1.0	25.2
2.5	45.7
5.0	62.6
7.5	71.3
10.0	76.6
15.0	82.7
20.0	85.6
30.0	88.7

Many experiments have shown that the coefficient of friction is dependent on the relative or *sliding velocity*.

$$V_S = \frac{V_W}{\cos\lambda}$$





Use curve *B* for high-quality materials
Use curve *A* when more friction is expected

~ 20% variations

Design Feasibility Report Submission

- Online at course website
- Before Sunday noon

Next class

- **Lab for Group 1**: Safety Training
- Friday 0800-1000, Oct 11
- Room 412, 5 Wisdom Valley

- **Discussion for Group 2**: Design Consultation
- Friday 0800-1000, Oct 11
- Room 202, 1 Lychee Park

Thank you!

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