ME303 Introduction to Mechanical Design

Lecture 06
Fatigue Failure Resulting from Variable Loading

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Agenda

Week 05, Wednesday, Oct 09, 2019

- Introduction to Fatigue in Metals
- Fatigue-Life Methods
  - Stress-Life | Strain-Life | Linear-Elastic Fracture Mechanics
- Fatigue Strength & the Endurance Limit
- Endurance Limit Modifying Factors
- Stress Concentration and Notch Sensitivity
- Example, Procedure and Solution
- Fluctuating Stresses

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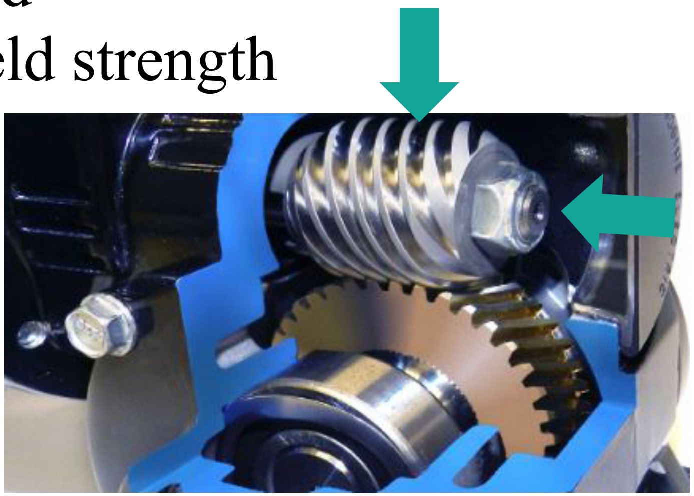
Introduction to Fatigue in Metals

The stresses vary with time or they fluctuate between different levels

- Fatigue Failure
  - Caused by the action of repeated or fluctuating stresses for a very large number of times
  - The actual maximum stress is observed to be well below the ultimate strength of the material, and
  - Quite frequently even below the yield strength



- Why so important?
  - Gives NO visible warning
  - Sudden and total, hence dangerous
  - Complicated phenomenon only partially understood

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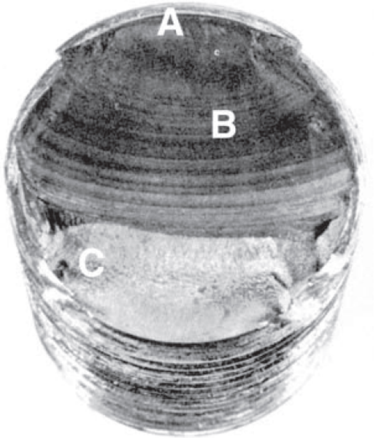
Three Stages of Fracture Development

How is Fatigue Failure different from Static Failure?

- Stage I
  - the initiation of ***one or more microcracks*** due to cyclic plastic deformation followed by crystallographic propagation extending from two to five grains about the origin.
  - not normally discernible to the naked eye.
- Stage II
  - progresses from microcracks to ***macrocracks*** forming parallel plateau-like fracture surfaces separated by longitudinal ridges.
- Stage III
  - occurs during the final stress cycle when the remaining material cannot support the loads, resulting in ***a sudden, fast fracture***.
  - Can be brittle, ductile, or a combination of both.

Fatigue failure of a bolt due to repeated unidirectional bending



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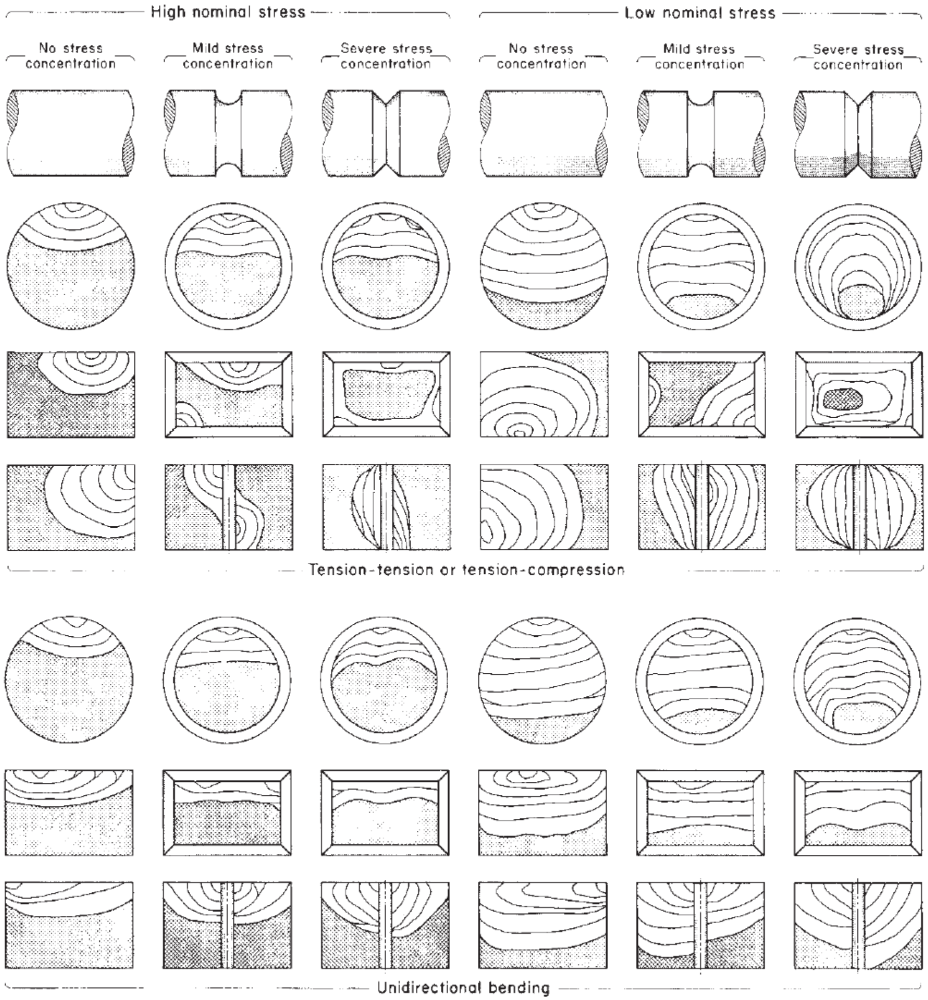
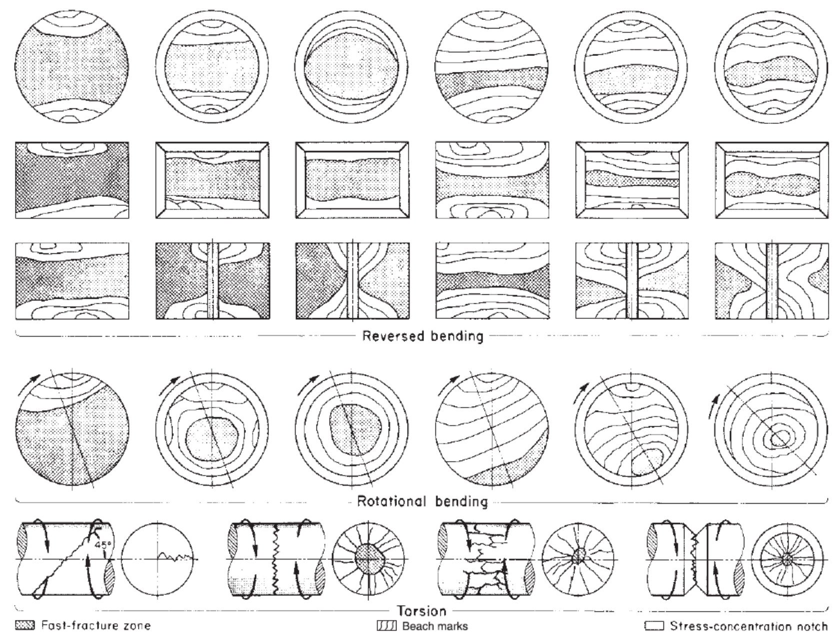
Schematics of Fatigue Fracture Surfaces

Fatigue failure is due to crack formation and propagation.



- A fatigue crack will typically initiate at a **discontinuity** in the material where the cyclic stress is a maximum.



Possible Causes of Discontinuities

The Engineering Reality

- Design of ***rapid changes*** in cross section, keyways, holes, etc. where stress concentrations occur.
- Elements that roll and/or slide ***against each other*** (bearings, gears, cams, etc.) ***under high contact pressure***, developing concentrated subsurface contact stresses that can cause surface pitting or spalling after many cycles of the load.
- ***Carelessness in locations*** of stamp marks, tool marks, scratches, and burrs; poor joint design; improper assembly; and other fabrication faults.
- ***Composition of the material itself*** as processed by rolling, forging, casting, extrusion, drawing, heat treatment, etc. Microscopic and submicroscopic surface and subsurface discontinuities arise, such as inclusions of foreign material, alloy segregation, voids, hard precipitated particles, and crystal discontinuities.

- Various conditions that can **accelerate crack initiation** include
  - residual tensile stresses, elevated temperatures, temperature cycling, a corrosive environment, and high frequency cycling.

An Example of Fatigue Failure

Drive Shaft fracture initiated at the end of the keyway.

Figure 6–3

Fatigue fracture of an AISI 4320 drive shaft. The fatigue failure initiated at the end of the keyway at points *B* and progressed to final rupture at *C*. The final rupture zone is small, indicating that loads were low. *(From ASM Handbook, Vol. 12:* Fractography, *2nd printing, 1992, ASM International, Materials Park, OH 44073-0002, fig 51, p. 120. Reprinted by permission of ASM International®, www.asminternational.org.)*

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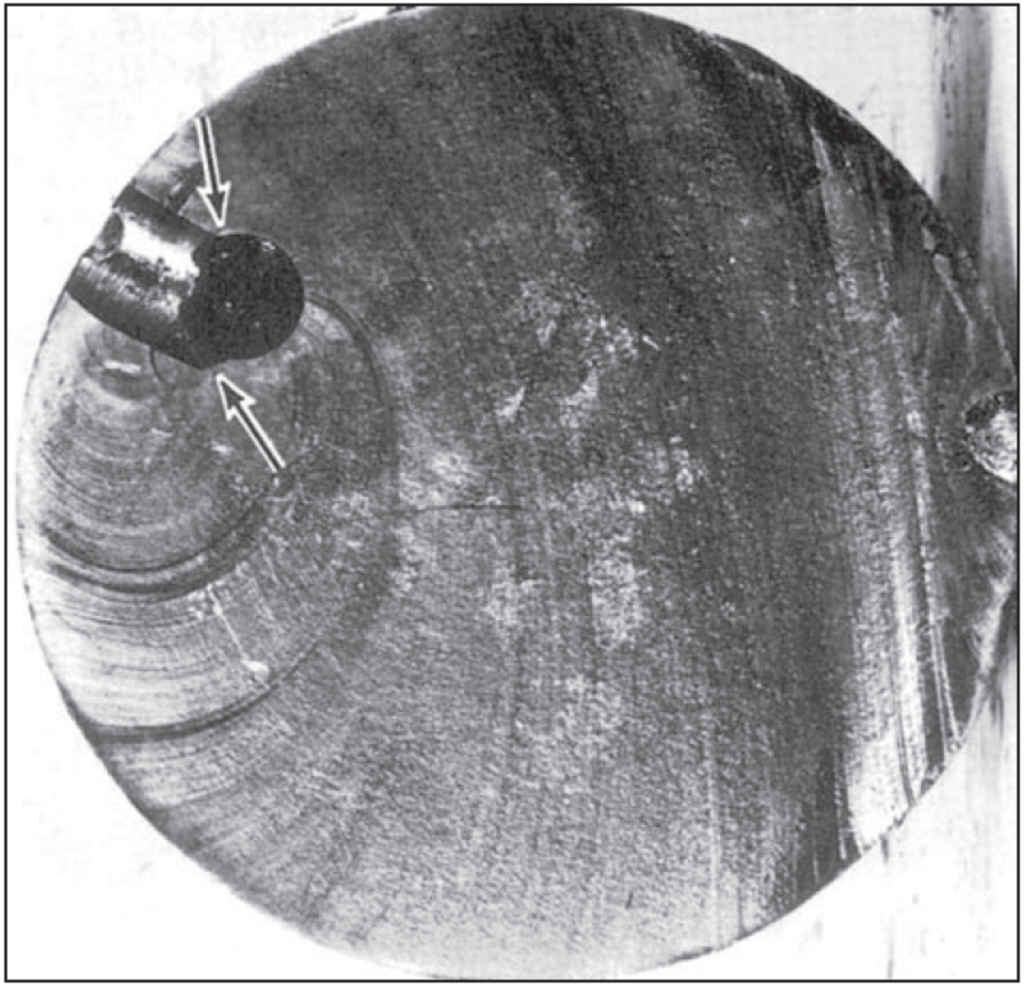
An Example of Fatigue Failure

Pin fracture initiated at the sharp corner of the grease hole.

Figure 6–4

Fatigue fracture surface of an AISI 8640 pin. Sharp corners of the mismatched grease holes provided stress concentrations that initiated two fatigue cracks indicated by the arrows. *(From ASM Handbook, Vol. 12:* Fractography, *2nd printing, 1992, ASM International, Materials Park, OH 44073-0002, fig 520, p. 331. Reprinted by permission of ASM International®, www.asminternational.org.)*



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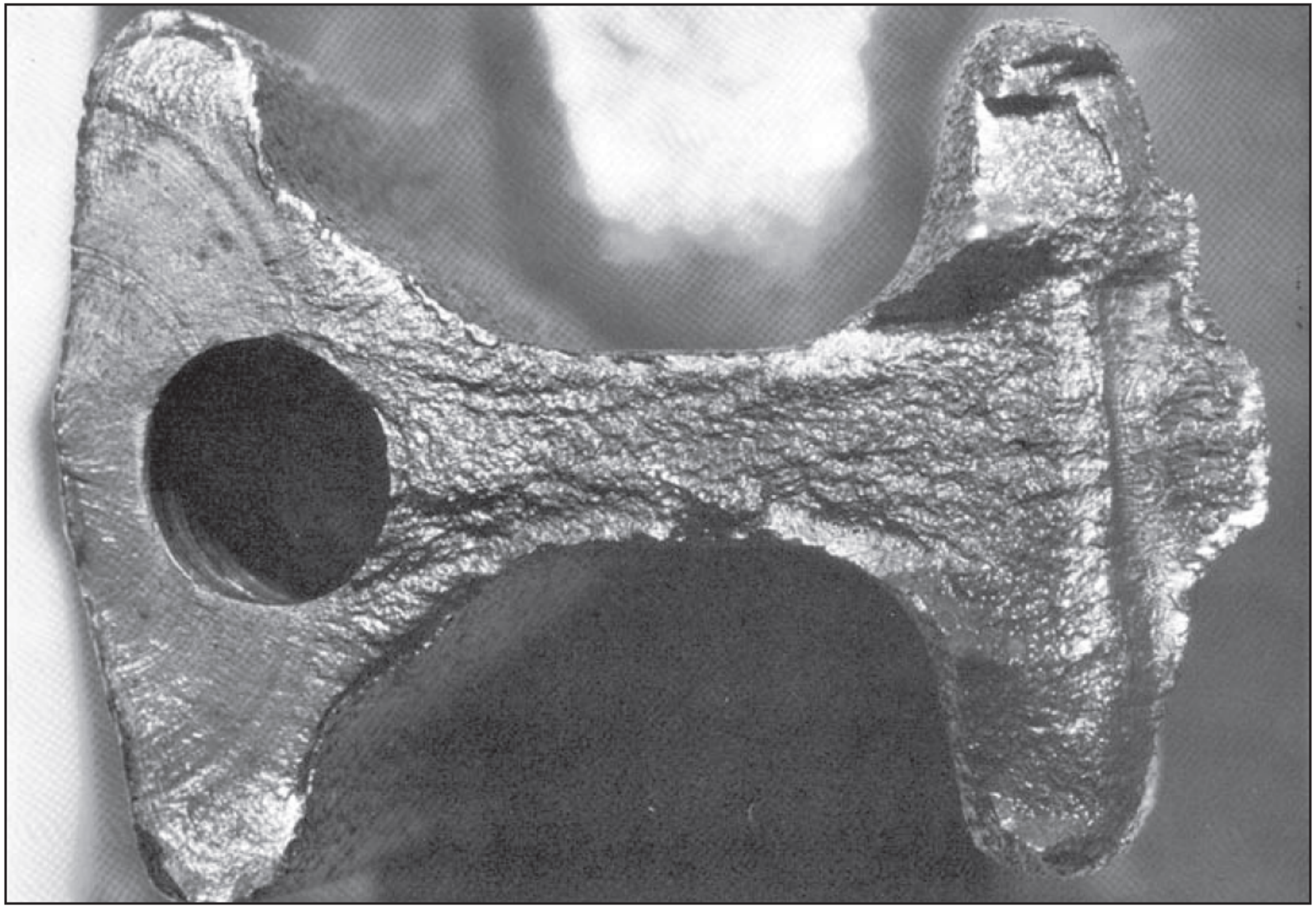
An Example of Fatigue Failure

A Forged Connection Rod fracture initiated at the left edge.

Figure 6–5

Fatigue fracture surface of a forged connecting rod of AISI 8640 steel. The fatigue crack origin is at the left edge, at the flash line of the forging, but no unusual roughness of the flash trim was indicated. The fatigue crack progressed halfway around the oil hole at the left, indicated by the beach marks, before final fast fracture occurred. Note the pronounced shear lip in the final fracture at the right edge. *(From ASM Handbook, Vol. 12:* Fractography, *2nd printing, 1992, ASM International, Materials Park, OH 44073-0002, fig 523, p. 332. Reprinted by permission of ASM International®, www.asminternational.org.)*

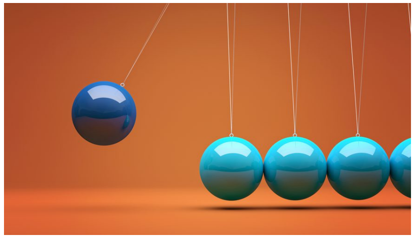


Approach to Fatigue Failure in Analysis and Design

A combination of Engineering and Science, but often Science fails to give a complete answer.

- Thus, while science has not yet completely explained the complete mechanism of fatigue, the engineer must still design things that will not fail.
  - Planes that fly safely;
  - Cars that are reliable and durable for use and profit.

- In a sense this is a classic example of the true meaning of engineering as contrasted with science.
  - Engineers use science to solve their problems if the science is available. But available or not, the problem must be solved, and whatever form the solution takes under these conditions is "called" engineering.
  - *Must be solved no matter what.*



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Fatigue-Life Methods

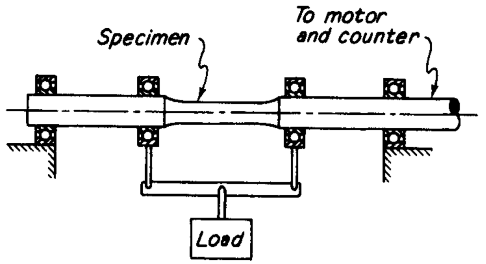
To predict the life in number of cycles to failure, N, for a specific level of loading

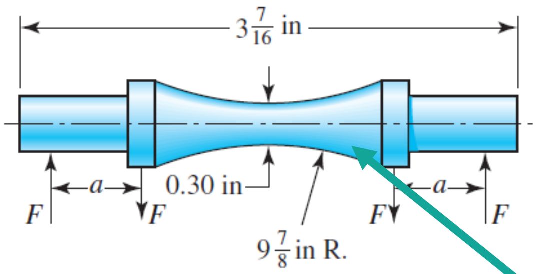
- The stress-life method
  - Based on stress levels only.
  - The **least accurate** approach, especially for low-cycle applications.
  - The **easiest to implement** for a wide range of design applications, has **ample supporting data**, and represents high-cycle applications adequately.
- The strain-life method
  - Involves **more detailed analysis** of the plastic deformation at localized regions where the stresses and strains are considered for life estimates.
  - Especially **good for low-cycle** fatigue applications.
  - Several idealizations are compounded, leading to **uncertainties** in the results.
- The fracture mechanics method
  - **Assumes a crack is already present and detected**.
  - To **predict crack growth** with respect to stress intensity.
  - Most practical when applied to **large structures** in conjunction with **computer** codes and a **periodic inspection** program.

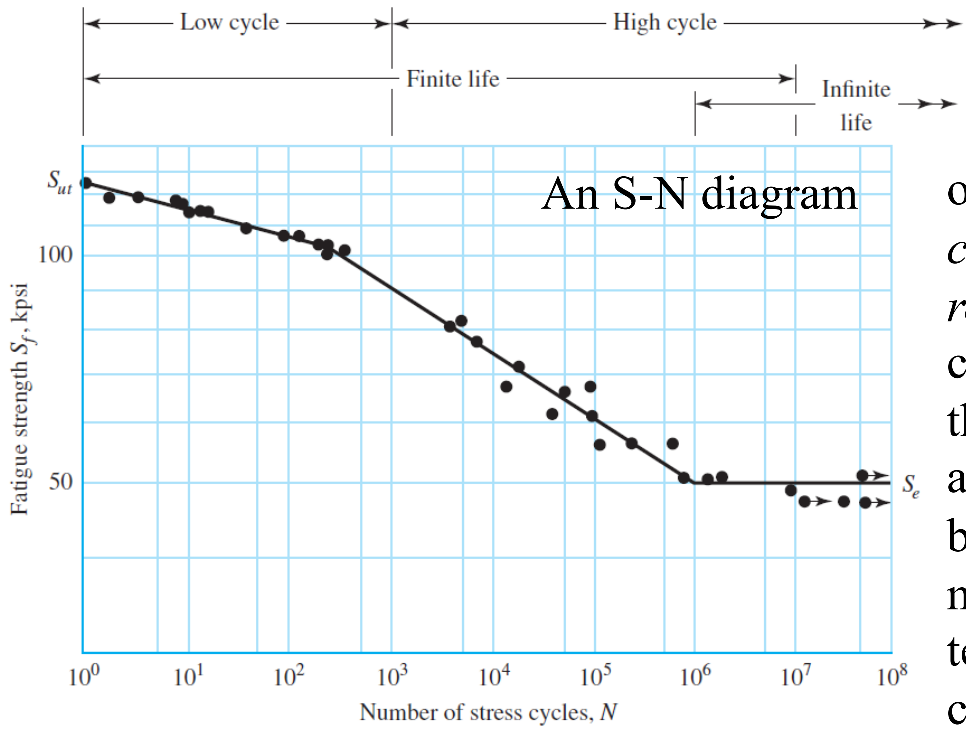
The Stress-Life Method

Specimens are subjected to repeated or varying forces of specified magnitudes while the cycles or stress reversals are counted to destruction

R. R. Moore high-speed rotating-beam machine







obtained by *completely reversed* stress cycles, in which the stress level alternates between equal magnitudes of tension and compression.

carefully machined and polished, with a final polishing in an axial direction to avoid circumferential scratches

N = 1 ?
N = 1/2 ?

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The Necessity for Testing

Engineering vs. Science

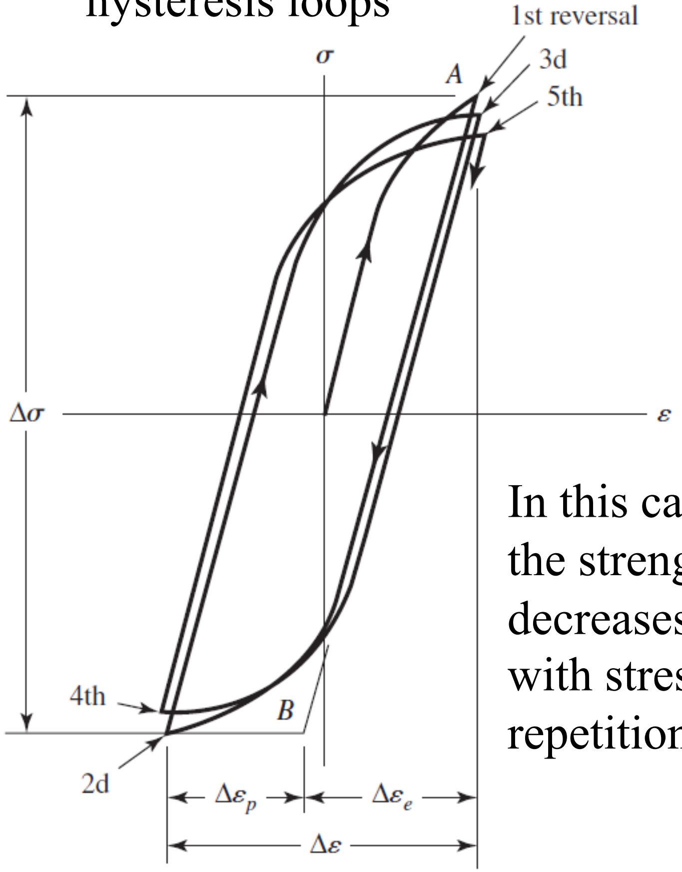
- It would really be **unnecessary** for us to proceed any further in the study of fatigue failure except for one important reason:
  - ***the desire to know why fatigue failures occur***
    - so that the most effective method or methods can be used to improve fatigue strength.
    - (so that we can guard against them in an optimum manner).

- The deterministic analysis presented in this chapter does not yield absolutely precise results.
  - The results should be taken as a **guide**,
    - as something that indicates **what is important** and **what is not important** in designing against fatigue failure.

The Strain-Life Method

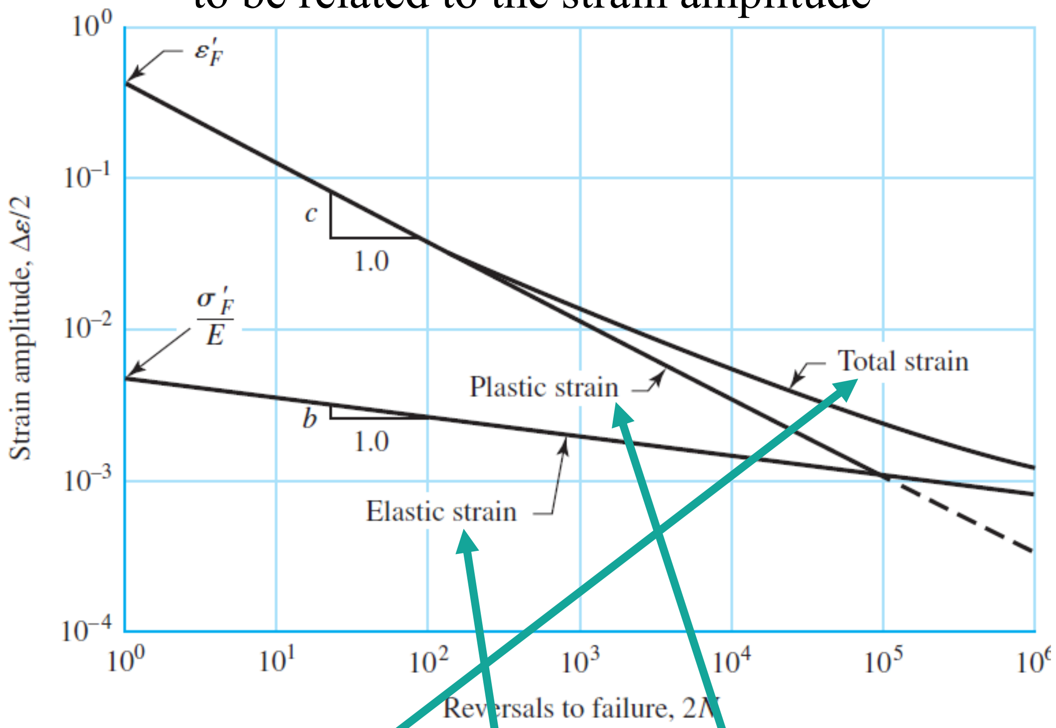
To explain the nature of fatigue failure, but of little use to design (lack of data).

True stress–true strain hysteresis loops



In this case, the strength decreases with stress repetitions.

the life in reversals to failure is found to be related to the strain amplitude



total-strain amplitude

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_F}{E}(2N)^b + \varepsilon'_F(2N)^c$$

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The Linear-Elastic Fracture Mechanics Method

Quantifying Crack Growth

Stress Intensity Range per cycle

$$\Delta K_I = \beta(\sigma_{max} - \sigma_{min})\sqrt{\pi a} = \beta \Delta\sigma \sqrt{\pi a}$$

$$\frac{da}{dN} = C(\Delta K_I)^m$$

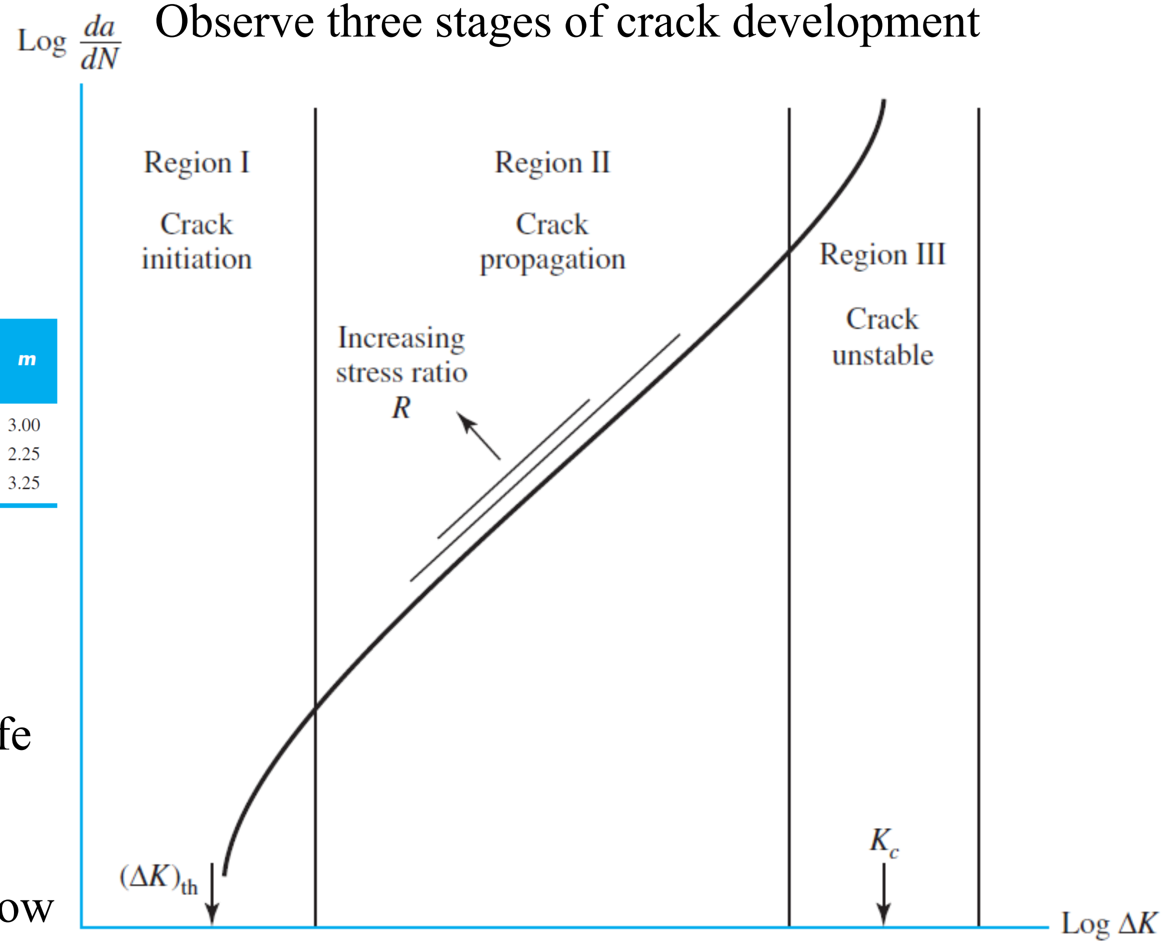
Material	C, $\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$	C, $\frac{\text{in/cycle}}{(\text{kpsi}\sqrt{\text{in}})^m}$	m
Ferritic-pearlitic steels	$6.89(10^{-12})$	$3.60(10^{-10})$	3.00
Martensitic steels	$1.36(10^{-10})$	$6.60(10^{-9})$	2.25
Austenitic stainless steels	$5.61(10^{-12})$	$3.00(10^{-10})$	3.25

$$\int_0^{N_f} dN = N_f = \frac{1}{C}\int_{a_i}^{a_f} \frac{da}{(\beta\Delta\sigma\sqrt{\pi a})^m}$$

Makes it convenient for numerical computation of the life

the threshold value below which a crack does not grow

Observe three stages of crack development



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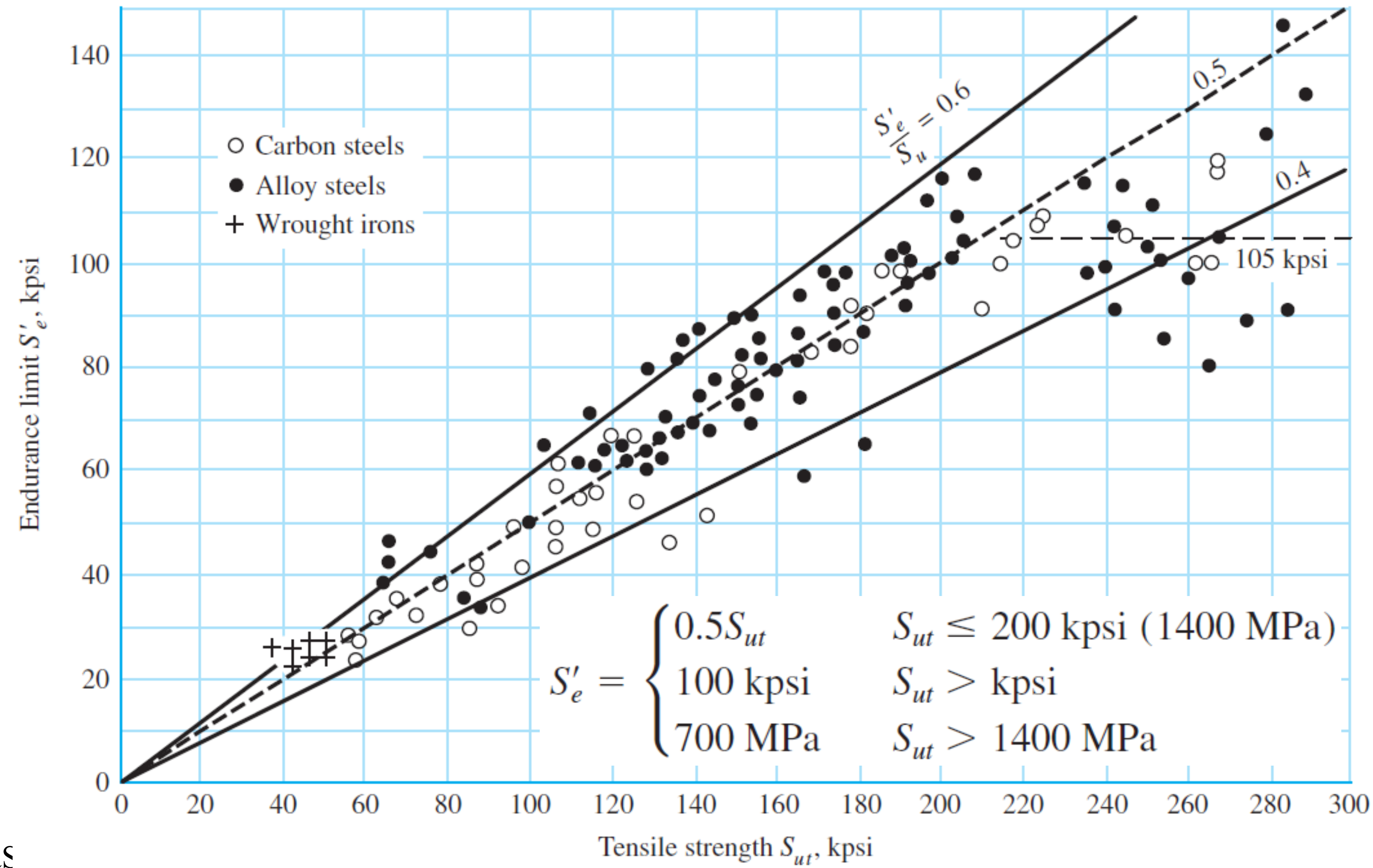
The Endurance Limit

Generally, stress testing is preferred to strain testing for endurance limits.



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Fatigue Strength

How do Engineers work with less information.

$$S_f = a\,N^b$$

Fatigue Life Constant

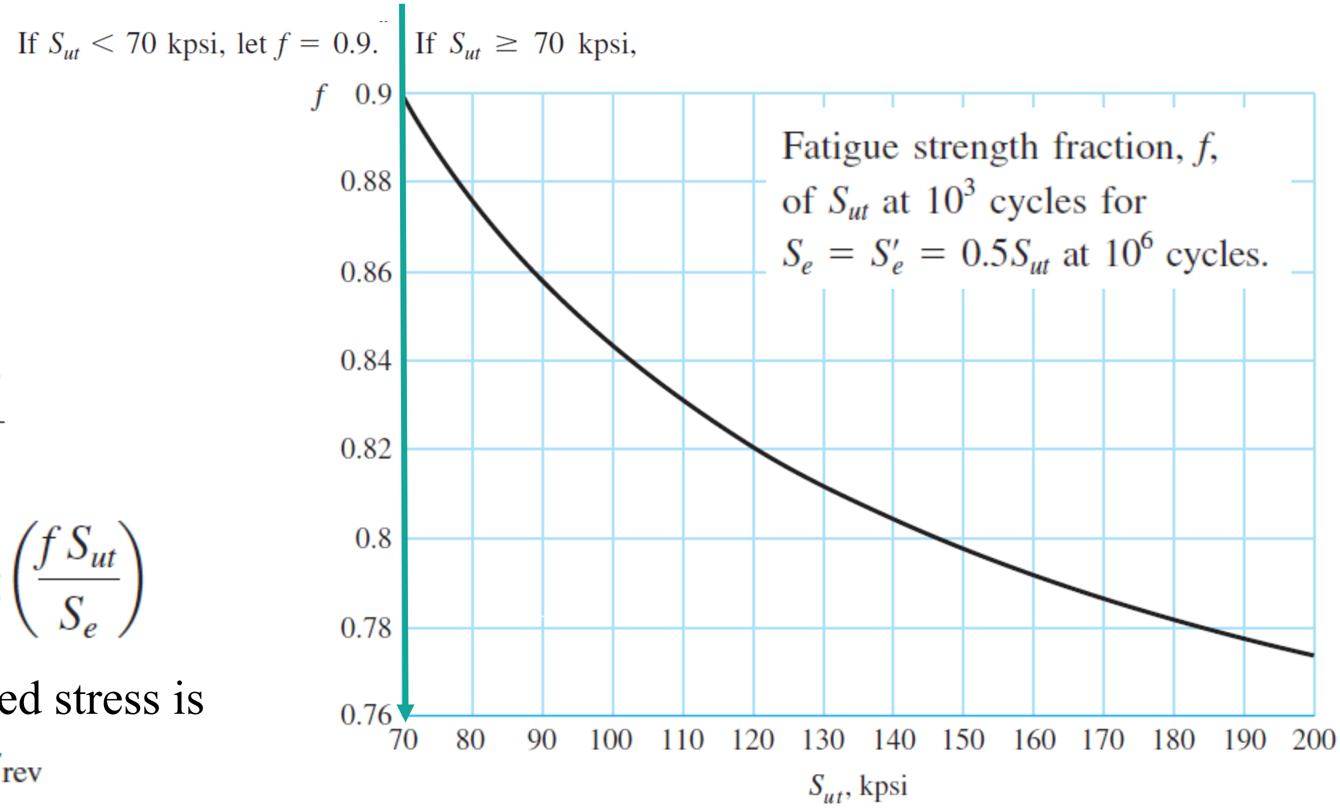
$$a = \frac{(f\,S_{ut})^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right)$$

If a completely reversed stress is given, setting $S_f = \sigma_{\text{rev}}$

$$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b}$$

Direct computation (estimate) of the life



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Endurance Limit Modifying Factors

A mismatch between the Perfect Experiment and the Changing Reality

- *Material:* composition, basis of failure, variability
- *Manufacturing:* method, heat treatment, fretting corrosion, surface condition, stress concentration
- *Environment:* corrosion, temperature, stress state, relaxation times
- *Design:* size, shape, life, stress state, speed, fretting, galling

- Marin's Estimation of Endurance Limit

$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$

surface condition modification factor

size modification factor

load modification factor

temperature modification factor

reliability factor

miscellaneous-effects modification factor

rotary-beam test specimen endurance limit

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Quantifying the Factors

Engineers' Solution

Surface Factor k_a

$$k_a = aS_{ut}^b$$

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

Size Factor k_b

For bending and torsion

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases}$$

For axial loading there is no size factor $k_b = 1$

Loading Factor k_c

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases}$$

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Temperature Factor k_d

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4$$

$$k_d = \frac{S_T}{S_{RT}}$$

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

Reliability Factor k_e

Corresponding to 8 Percent Standard Deviation of the Endurance Limit

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

Miscellaneous-Effects Factor k_f

not always available

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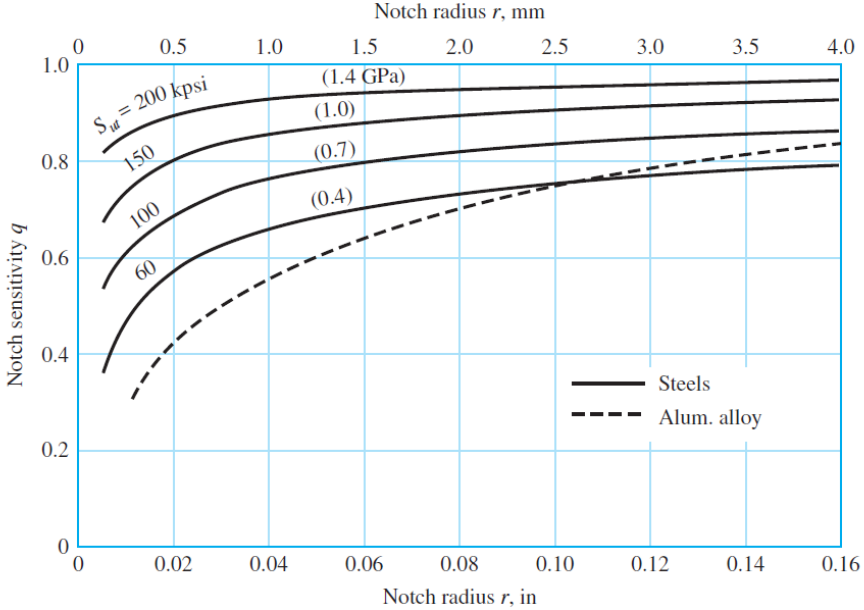
Stress Concentration and Notch Sensitivity

Some materials are not fully sensitive to the presence of notches

- Fatigue Stress-Concentration Factor K_f
  - A reduced value of the stress concentration factor K_t

$$K_f = \frac{\text{maximum stress in notched specimen}}{\text{stress in notch-free specimen}}$$



- Notch sensitivity q

$$q = \frac{K_f - 1}{K_t - 1} \quad \text{or} \quad q_{\text{shear}} = \frac{K_{fs} - 1}{K_{ts} - 1}$$

- **Put in use**

$$K_f = 1 + q(K_t - 1)$$

$$K_{fs} = 1 + q_{\text{shear}}(K_{ts} - 1)$$

Or use Neuber equation

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} \qquad q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

Bending or axial: $\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$ (6–35a)

Torsion: $\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$ (6–35b)

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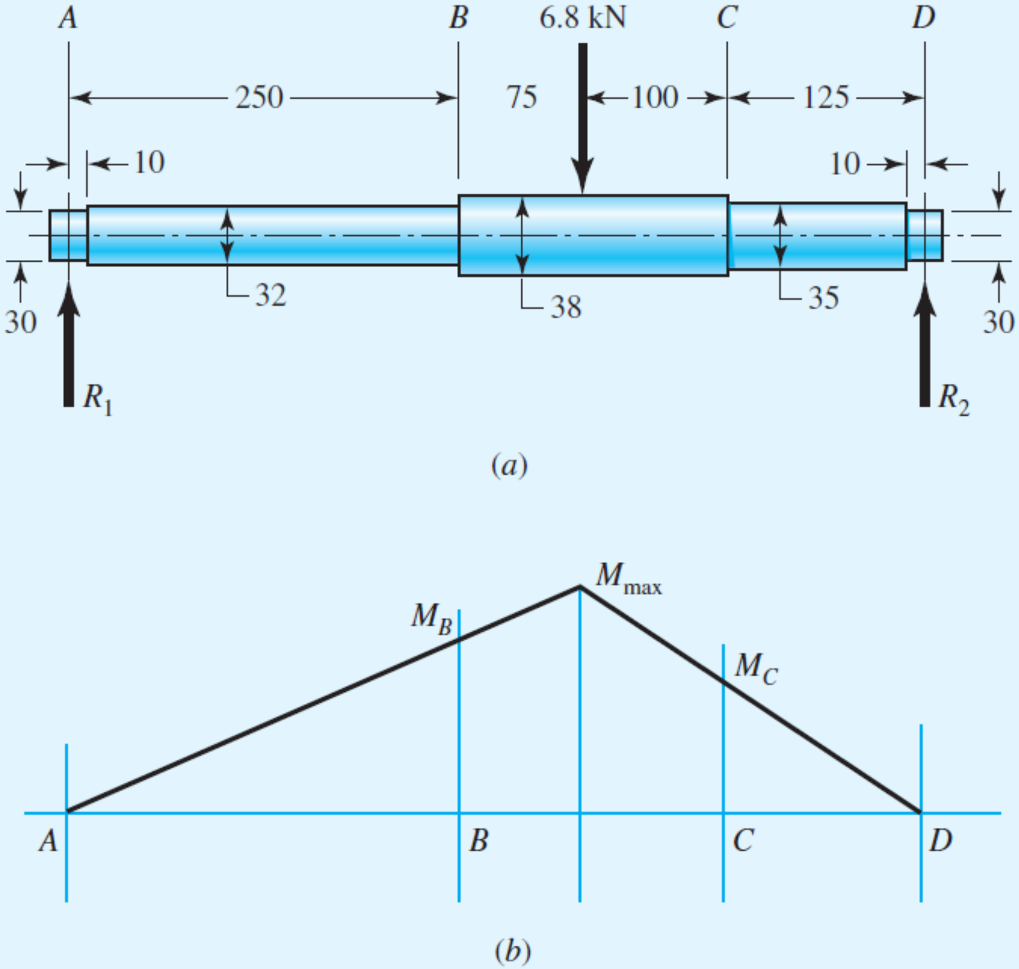
Example: Estimate the Life of a Part

A rotating shaft simply supported in ball bearings at A and D and loaded by a nonrotating force F of 6.8 kN (Use ASTM "minimum" strength)

Figure 6–22

(a) Shaft drawing showing all dimensions in millimeters; all fillets 3-mm radius. The shaft rotates and the load is stationary; material is machined from AISI 1050 cold-drawn steel. (b) Bending-moment diagram.



failure will probably occur at ***B*** *rather than at* ***C*** *or at the point of* ***maximum moment****.*

1 Determine S'_e either from test data or

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \leq 200 \text{ kpsi } (1400 \text{ MPa}) \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$

2 Modify S'_e to determine S_e.

$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$

3 Determine fatigue stress-concentration factor, K_f or K_{fs}.

4 Apply K_f or K_{fs} by *either* dividing S_e by it *or* multiplying it with the purely reversing stress, *not* both.

5 Determine fatigue life constants a and b. If $S_{ut} \geq 70$ kpsi, determine f from Fig. 6–18, p. 293. If $S_{ut} < 70$ kpsi, let $f = 0.9$.

$$a = (f S_{ut})^2 / S_e$$

$$b = -[\log (f S_{ut}/S_e)]/3$$

6 Determine fatigue strength S_f at N cycles, or, N cycles to failure at a reversing stress σ_{rev}

(*Note*: this only applies to purely reversing stresses where $\sigma_m = 0$).

$$S_f = aN^b$$

$$N = (\sigma_{\text{rev}}/a)^{1/b}$$

Solution Procedure

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Solution: for the **Endurance Limit**

Look up the tables…

- Estimate the endurance limit $S'_e = 0.5(690) = 345\ \text{MPa}$
- Determine the factors

$$k_a = 4.51(690)^{-0.265} = 0.798$$

$$k_b = (32/7.62)^{-0.107} = 0.858$$

$$k_c = k_d = k_e = k_f = 1$$

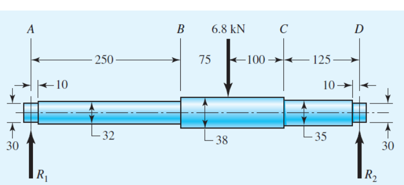
$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$

$$S_e = 0.798(0.858)345 = 236\ \text{MPa}$$

Table A–20

Deterministic ASTM Minimum Tensile and Yield Strengths for Some Hot-Rolled (HR) and Cold-Drawn (CD) Steels [The strengths listed are estimated ASTM minimum values in the size range 18 to 32 mm ($\frac{3}{4}$ to $1\frac{1}{4}$ in). These strengths are suitable for use with the design factor defined in Sec. 1–10, provided the materials conform to ASTM A6 or A568 requirements or are required in the purchase specifications. Remember that a numbering system is not a specification.] *Source:* 1986 SAE Handbook, p. 2.15.

1 UNS No.	2 SAE and/or AISI No.	3 Process-ing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10500	1050	HR	620 (90)	340 (49.5)	15	35	179
		CD	690 (100)	580 (84)	10	30	197



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Solution: for the
Fatigue Stress-Concentration Factor

Look up the tables …

- $D/d = \frac{38}{32} = 1.1875$
- $r/d = \frac{3}{32} = 0.09375$
- Read $K_t = 1.65$
- Calculate
  - $\sqrt{a} = 0.313\sqrt{\text{mm}}$
- Then Fatigue Factor

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}}$$

$$= 1 + \frac{1.65 - 1}{1 + 0.313/\sqrt{3}} = 1.55$$

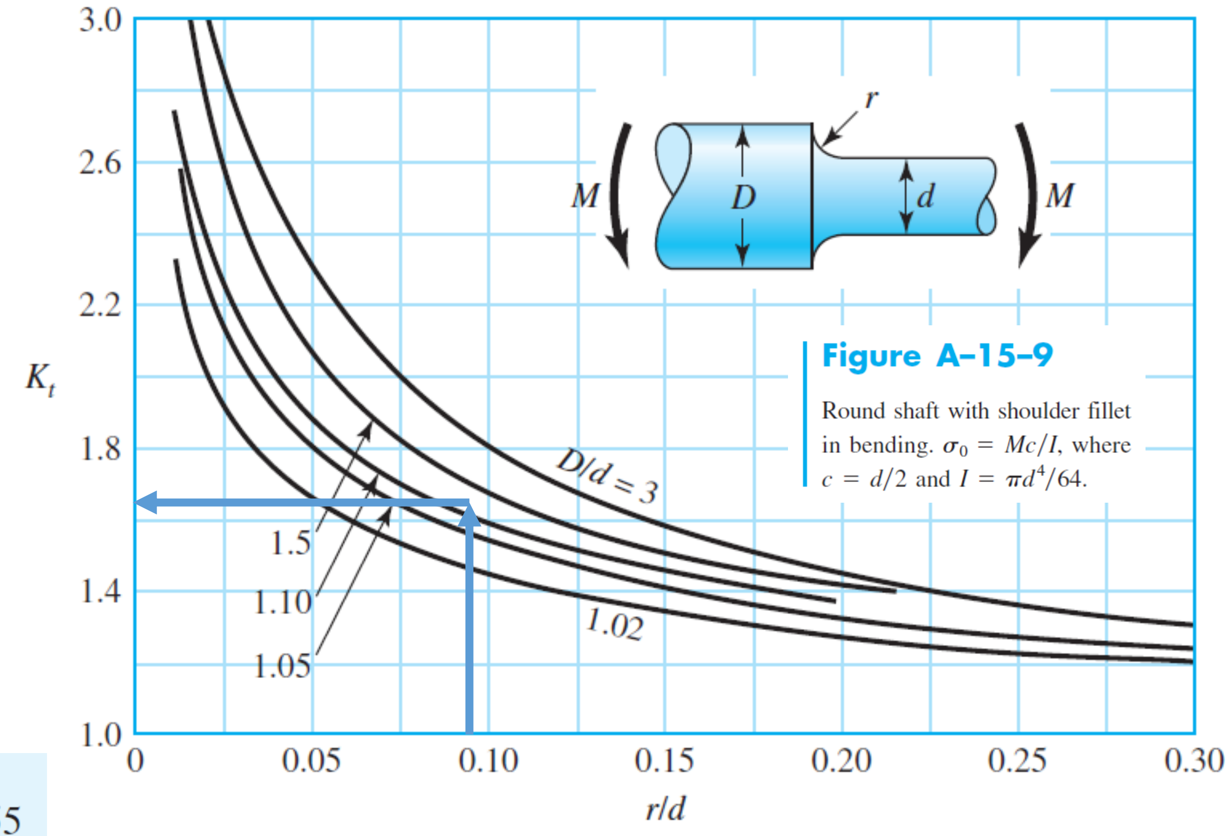


Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

Bending or axial: $\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$

Solution: for the First Cycle

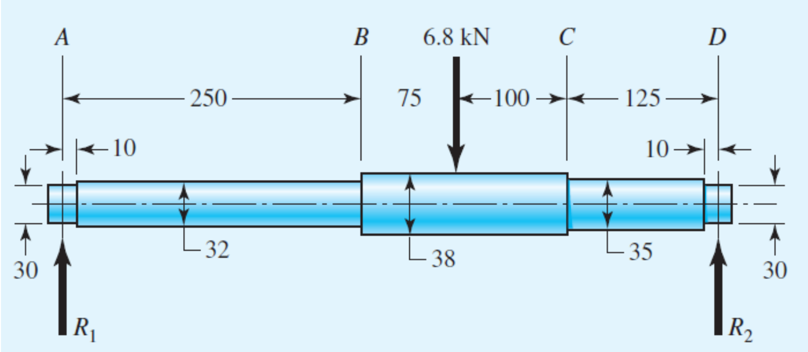
Look up the tables …

- Bending moment at B

$$M_B = R_1 x = \frac{225F}{550}250 = \frac{225(6.8)}{550}250 = 695.5\ \text{N}\cdot\text{m}$$

- Section modulus to the left of B

$$I/c = \pi d^3/32 = \pi 32^3/32 = 3.217\ (10^3)\ \text{mm}^3.$$



- Assuming infinite life, the reversing bending moment

$$\sigma_{\text{rev}} = K_f \frac{M_B}{I/c} = 1.55\frac{695.5}{3.217}(10)^{-6} = 335.1(10^6)\ \text{Pa} = 335.1\ \text{MPa}$$

  - Greater than Endurance Limit, less than Yield Limit
    - Meaning we have both finite life and no yielding on the first cycle

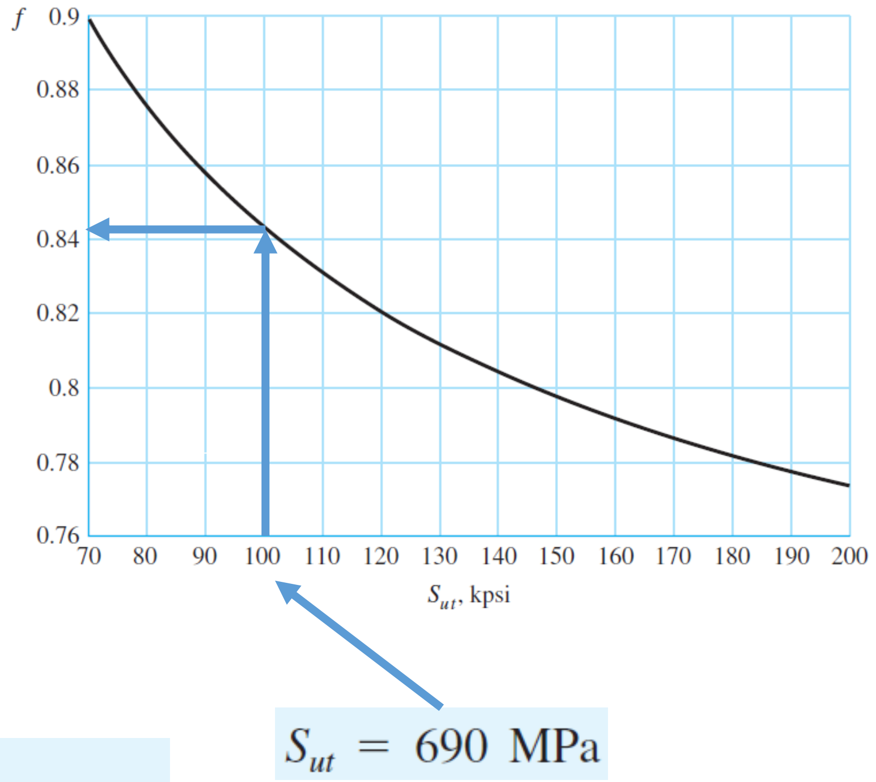
Solution: for the **Life Cycles**

Look up the tables …

- For finite life, $f = 0.844$

$$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.844(690)]^2}{236} = 1437 \text{ MPa}$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left[\frac{0.844(690)}{236}\right] = -0.1308$$



- Finally, calculate the Life Cycle

$$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{335.1}{1437}\right)^{-1/0.1308} = 68(10^3) \text{ cycles}$$

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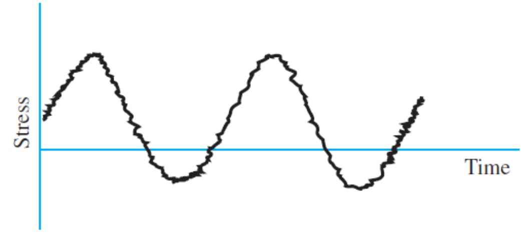
Characterizing Fluctuating Stresses

The Maximum, the Minimum and the Patterns

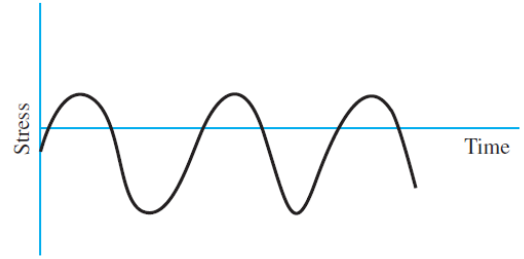
$$F_m = \frac{F_{\max} + F_{\min}}{2}$$

$$F_a = \left| \frac{F_{\max} - F_{\min}}{2} \right|$$

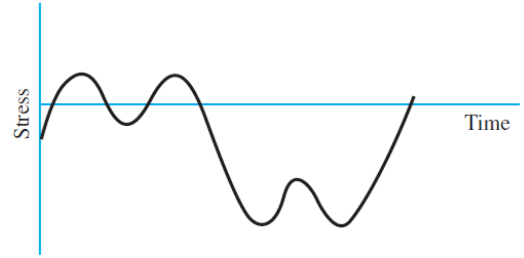
$\sigma_{\min}$ = minimum stress
$\sigma_{\max}$ = maximum stress
σ_a = amplitude component
σ_m = midrange component
σ_r = range of stress
σ_s = static or steady stress



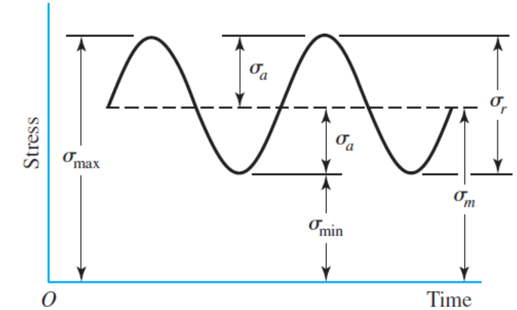
fluctuating stress with high frequency ripple



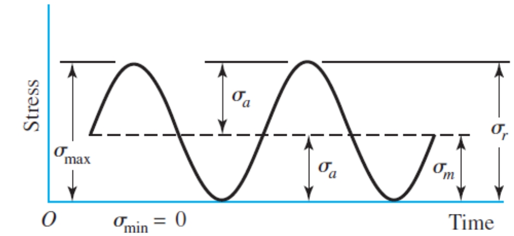
Non-sinusoidal fluctuating stress



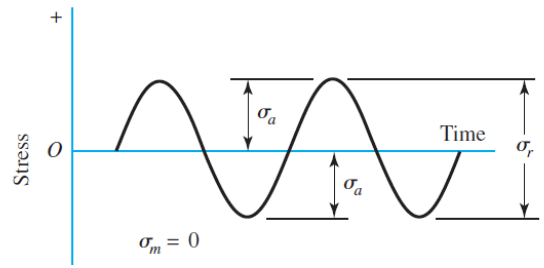
Non-sinusoidal fluctuating stress



Sinusoidal fluctuating stress



Repeated stress



completely reversed sinusoidal stress

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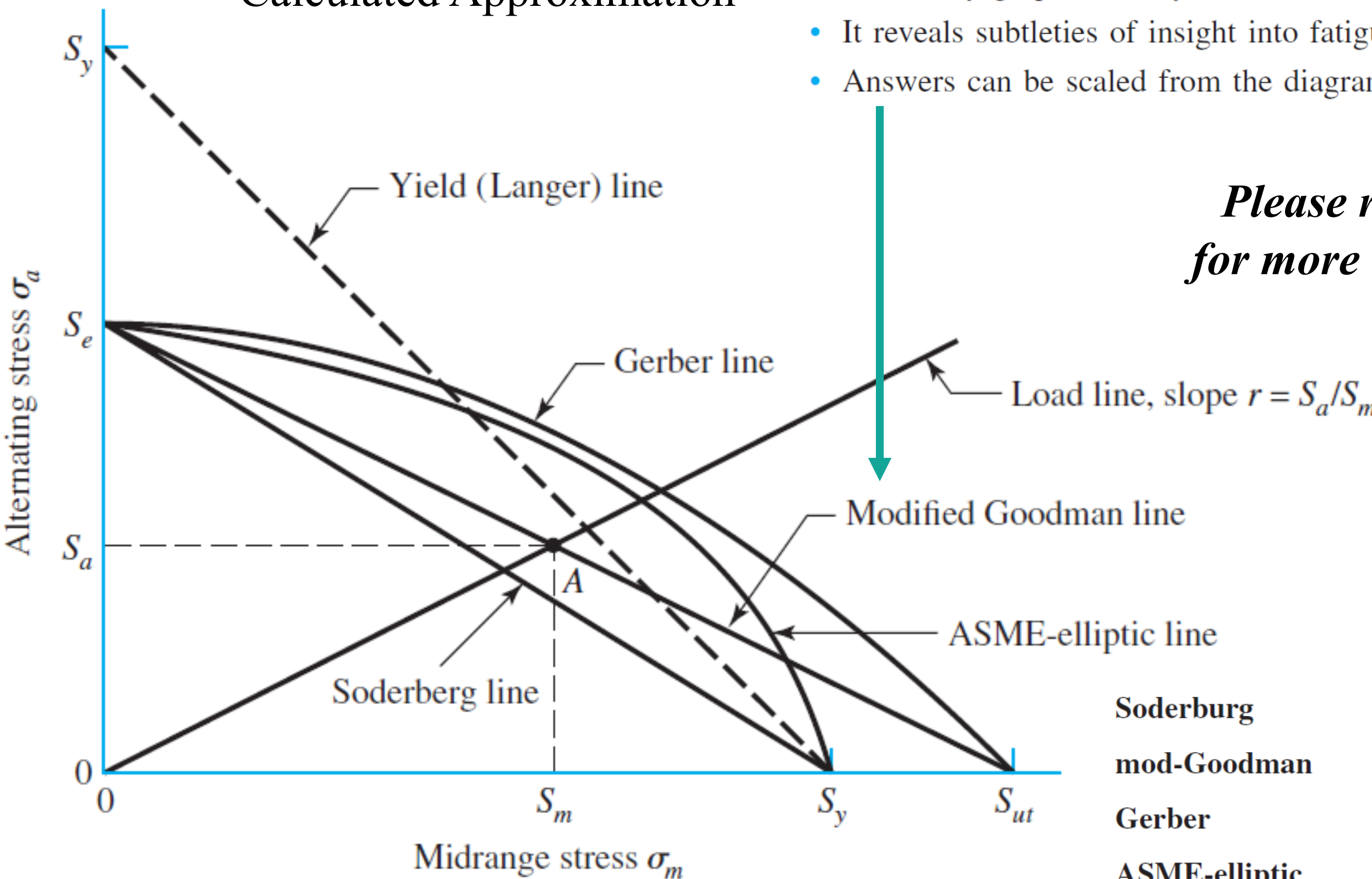
Fatigue Failure Criteria for Fluctuating Stress

Soderberg | mod-Goodman | Gerber | ASME-elliptic | Langer static yield

Different Assumptions for Calculated Approximation

- It is a straight line and the algebra is linear and easy.
- It is easily graphed, every time for every problem.
- It reveals subtleties of insight into fatigue problems.
- Answers can be scaled from the diagrams as a check on the algebra.



Please refer to the textbook for more details if interested.

Soderburg	$\sigma_a/S_e + \sigma_m/S_y = 1/n$
mod-Goodman	$\sigma_a/S_e + \sigma_m/S_{ut} = 1/n$
Gerber	$n\sigma_a/S_e + (n\sigma_m/S_{ut})^2 = 1$
ASME-elliptic	$(\sigma_a/S_e)^2 + (\sigma_m/S_y)^2 = 1/n^2$

Next class

- **Lab for Group 1**: Mechanism Design
- Friday 0800-1000, Oct 11
- Room 412, 5 Wisdom Valley

- **Discussion for Group 2**: Mechanism Design
- Friday 0800-1000, Oct 11
- Room 202, 1 Lychee Park

The class after the next

- **Groups 1+2**: Joint Design Project
- Project Briefing & Design Ideation
- ***Lab report submission before noon***

- Friday 0800-1000, Oct 12
- Room 202, 1 Lychee Park

Thank you!

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