ME303 Introduction to Mechanical Design

Lecture 05
Failures Resulting from Static Loading

Song Chaoyang

Assistant Professor

Department of Mechanical and Energy Engineering

songcy@sustech.edu.cn

SUSTech

Southern University of Science and Technology

Project Teams of 8

Students not yet have a team can choose one from below and contact the team leader. Starting from the team with the least members.

Group 1-Team 1

- Biao Wu 11710219@mail.sustech.edu.cn
- Kai Lu 11710915@mail.sustech.edu.cn
- Yuxuan Chen 11712312@mail.sustech.edu.cn
- Zicong Zhang 11711203@mail.sustech.edu.cn
- Ziqi Liu 11712526@mail.sustech.edu.cn

Group 1-Team 2

- Kaiwen Fang 11711804@mail.sustech.edu.cn
- Peilun Zeng 11711901@mail.sustech.edu.cn
- Shijian Wu 11710826@mail.sustech.edu.cn
- Shilong Yao 11711721@mail.sustech.edu.cn
- Xinyu Hu 11711530@mail.sustech.edu.cn

Group 1-Team 3

- Chengsheng Huang 11710909@mail.sustech.edu.cn
- Haotian Cui 11710703@mail.sustech.edu.cn
- Haoyi Lv 11612506@mail.sustech.edu.cn
- Jin Lu 11710635@mail.sustech.edu.cn
- Qingyang Wang 11711022@mail.sustech.edu.cn

Group 1-Team 4

- Hengrui Shang 11610134@mail.sustech.edu.cn
- Jie Cheng 11710412@mail.sustech.edu.cn
- Shuxin Zheng 11610613@mail.sustech.edu.cn
- Yonglin Jing 11712605@mail.sustech.edu.cn

Group 2-Team 5

- Huizhao Ding 11712803@mail.sustech.edu.cn
- Ruoyu Sun 11712201@mail.sustech.edu.cn
- Shuyuan Ma 11710538@mail.sustech.edu.cn
- Xiaodi Sui 11712839@mail.sustech.edu.cn
- Yanjing Liu 11711036@mail.sustech.edu.cn

Group 2-Team 6

- Haowen Wang 11711020@mail.sustech.edu.cn
- Shaoqian Li 11710320@mail.sustech.edu.cn
- Zhengtao Liu 11711018@mail.sustech.edu.cn
- Zhun Huang 11711606@mail.sustech.edu.cn
- Zhuotong Yu 11712101@mail.sustech.edu.cn

Group 2-Team 7

- Haiou Liao 11610605@mail.sustech.edu.cn
- Qingshuang Lu 11710435@mail.sustech.edu.cn
- Qixiao Liu 11711810@mail.sustech.edu.cn
- Shaofei Wang 11710513@mail.sustech.edu.cn
- Xuanwu Chen 11710323@mail.sustech.edu.cn

Group 2-Team 8

- Hui Li 11712731@mail.sustech.edu.cn
- Jixiang Qiao 11710817@mail.sustech.edu.cn
- Leo Wang 11711612@mail.sustech.edu.cn
- Xiansen Yang 11710218@mail.sustech.edu.cn

AncoraSIR.com

SUSTech Southern University of Science and Technology

Agenda

Week 04, Wednesday, Sep 25, 2019

- Basic Concepts
  - Static Strength
  - Cartesian Stress Components
  - Mohr's Circle for Plane Stress
  - General 3D Stress
  - Stress Concentration
- Failure Theories for Ductile Materials
  - Maximum Shear Stress (MSS)
  - Distortion Energy (DE)
  - Ductile Coulomb-Mohr (DCM)
- Failure Theories for Brittle Materials
  - Maximum Normal Stress (MNS)
  - Brittle Coulomb-Mohr (BCM)
  - Modified Mohr (MM)
- Selection of Failure Criteria
- Introduction to Fracture Mechanics

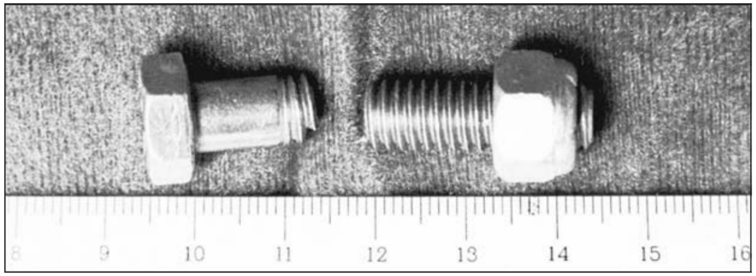
AncoraSIR.com

Why Study "Failure"?

The predictability of permanent distortion or separation.

- The strengths of the <u>*mass-produced parts*</u> will all be somewhat different from the others in the collection or ensemble
  - Variations in dimensions, machining, forming, and composition.

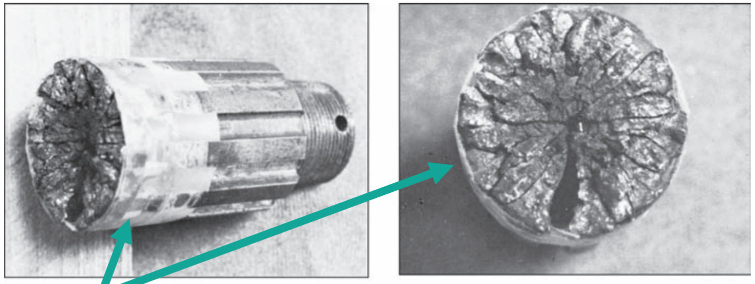
Failure of an overhead-pulley retaining bolt on a weightlifting machine.



- Static Load
  - A stationary force or couple applied to a member.
  - Cannot change in any manner.

- "Failure"
  - can mean a part has separated into two or more pieces;
  - has become permanently distorted, thus ruining its geometry;
  - has had its reliability downgraded;
  - or has had its function compromised.

Failure of a truck drive-shaft spline due to corrosion fatigue.



- In strength-sensitive situations, the designer must
  - separate *mean stress* and *mean strength*
  - at the *critical location* sufficiently

AncoraSIR.com

Clear tape is used to hold the pieces in place.

Southern University of Science and Technology

Static Strength

The Engineering Reality of COST

- *Ideally*, in designing any machine element, the engineer should have available the results of a great many strength tests of the particular material chosen.
  - These tests should be made on specimens having the *same* heat treatment, surface finish, and size as the element the engineer proposes to design; and
  - the tests should be made under *exactly the same* loading conditions as the part will experience in service.

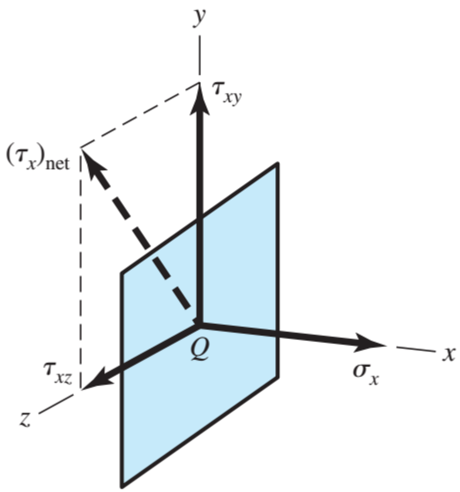
- Four Design Categories => *Use Only Published Values to Design (Look up the Tables)*
  1. Failure of the part would endanger human life, or the part is made in extremely large quantities; consequently, **an elaborate testing program** is justified during design.
  2. The part is made in large enough quantities that **a moderate series of tests** is feasible.
  3. The part is made in such small quantities that **testing is not justified at all**; or the design must be completed so rapidly that there is **not enough time for testing**.
  4. The part has already been designed, manufactured, and tested and found to be unsatisfactory. Analysis is required **to understand why** the part is unsatisfactory and what to do to improve it.

AncoraSIR.com

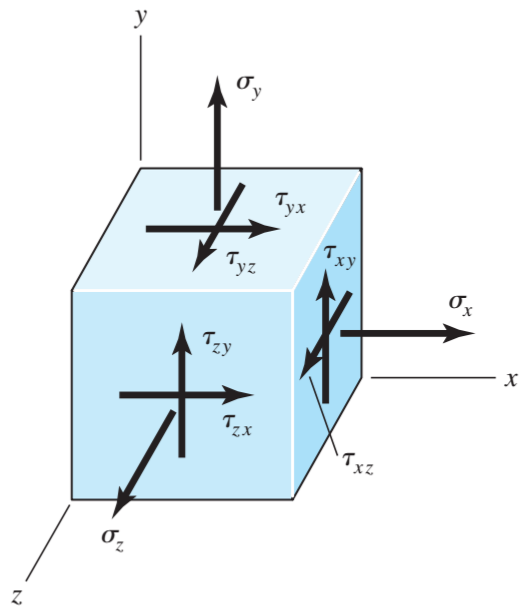
SUSTech Southern University of Science and Technology

Cartesian Stress Components

Normal Stress σ | Shear Stress τ

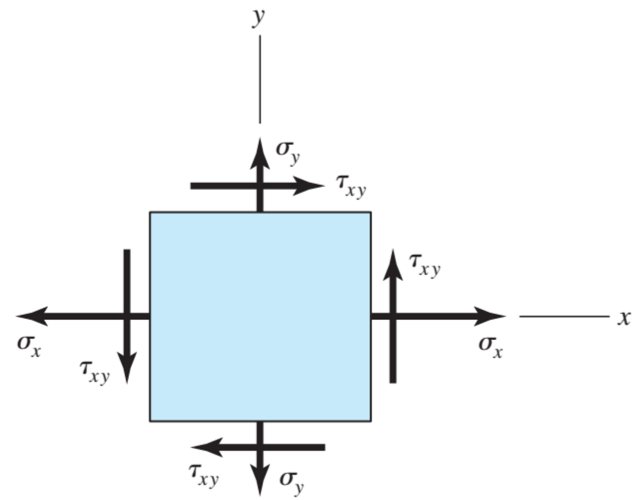


an infinitesimal surface area isolation at a point Q within a body where the surface normal is the x direction.



σ_x, σ_y, σ_z, τ_{xy}, τ_{xz},

τ_{yx}, τ_{yz}, τ_{zx}, and τ_{zy}.



Plane Stress occurs when the stresses on one surface are zero.

$$\sigma_z = \tau_{zx} = \tau_{zy} = 0.$$

$$\tau_{yx} = \tau_{xy}, \text{ and } \tau_{yz} = \tau_{zy} = \tau_{xz} = \tau_{zx} = 0.$$

The first subscript indicates the direction of the surface normal whereas the second subscript is the direction of the shear stress.

AncoraSIR.com

Plane-Stress Transformation Equations

Finding the Max/Min Normal/Shear Stresses & Their Principal Planes

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\phi + \tau_{xy}\sin 2\phi$$

maximizes σ → the max/min normal stresses

$$\tan 2\phi_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

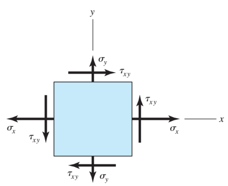
$$\tau = -\frac{\sigma_x - \sigma_y}{2}\sin 2\phi + \tau_{xy}\cos 2\phi$$

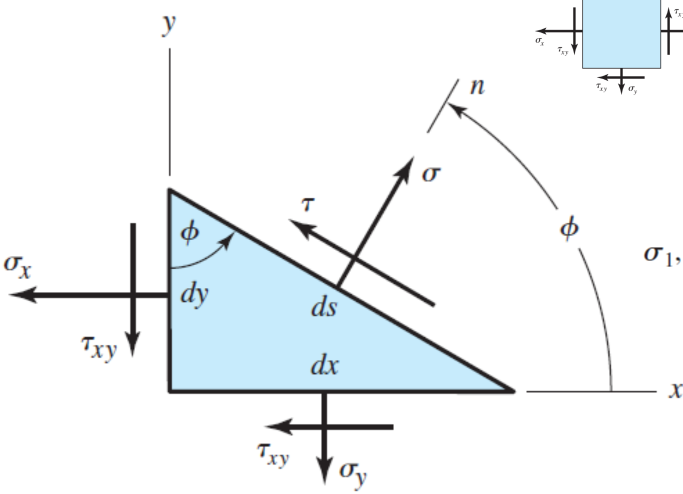
$\tau = 0$. The two surfaces containing principal stresses have **zero shear stresses**

maximizes τ

$$\tan 2\phi_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$2\phi_s$ and $2\phi_p$ are angles 90° apart





the max/min shear stresses

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_1, \tau_2 = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

the **two** surfaces containing the maximum shear stresses also contain **equal normal stresses**

$$\sigma = \frac{\sigma_x + \sigma_y}{2}$$

But there are always two of them …

AncoraSIR.com

SUSTech
Southern University of Science and Technology

Mohr's Circle Shear Convention

A Graphical Representation of Stress State at a *single* point in a structure

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\phi + \tau_{xy}\sin 2\phi$$

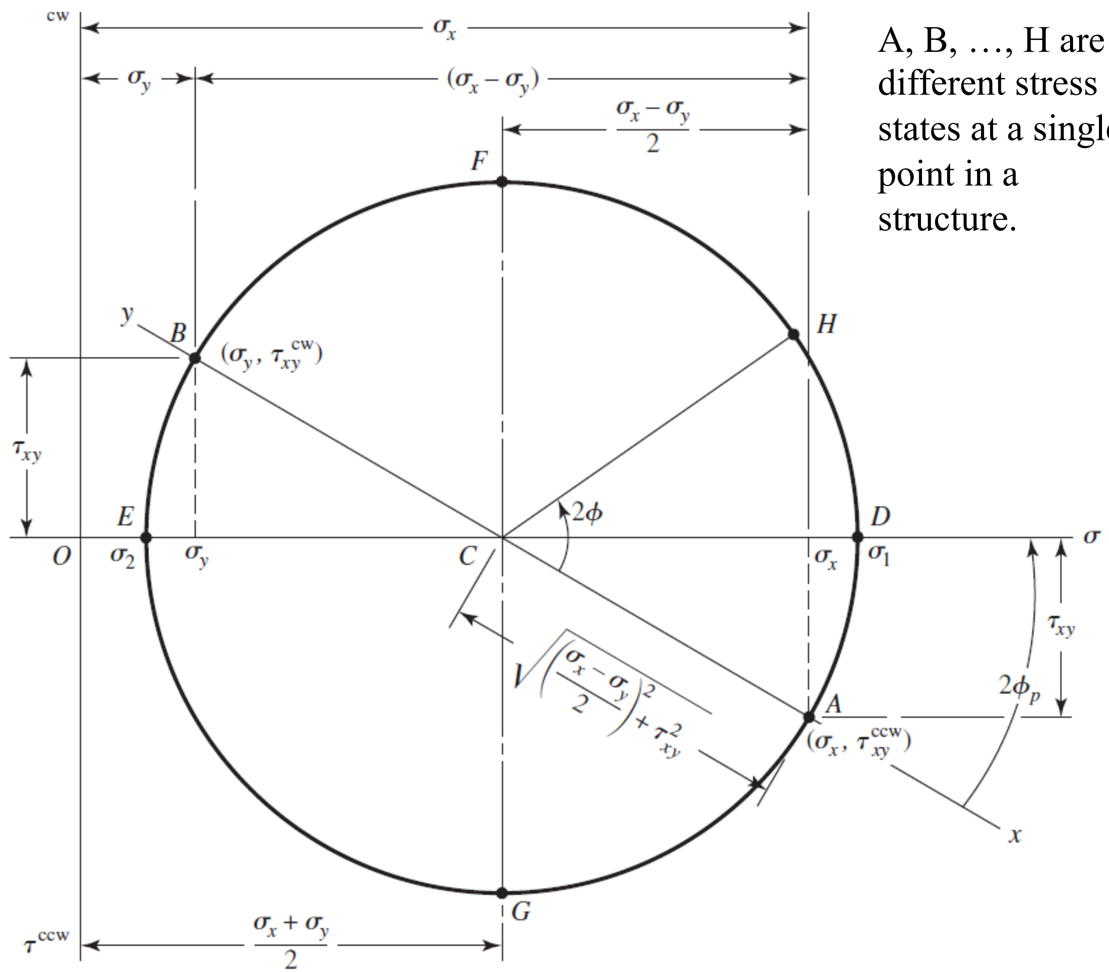
$$\tau = -\frac{\sigma_x - \sigma_y}{2}\sin 2\phi + \tau_{xy}\cos 2\phi$$

- Mohr's Circle
  - a circle plotted in the σ, τ plane

$$C = (\sigma, \tau) = [(\sigma_x + \sigma_y)/2, 0]$$

$$R = \sqrt{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2}.$$

- Convention
  - Shear stresses tending to rotate the element clockwise (cw) are plotted *above* the σ axis.
  - Shear stresses tending to rotate the element counterclockwise (ccw) are plotted *below* the σ axis.



A, B, …, H are different stress states at a single point in a structure.

AncoraSIR.com

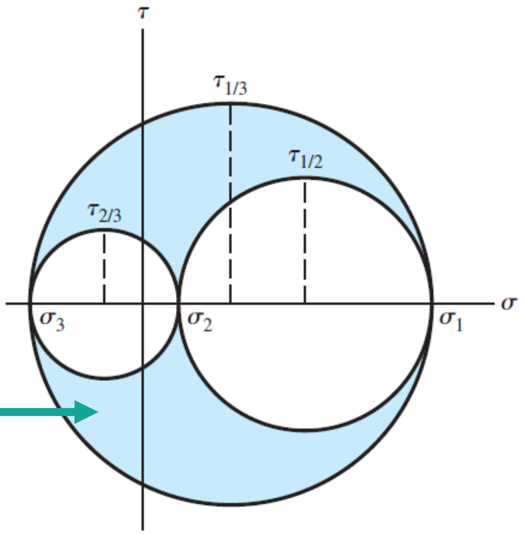
General 3D Stress

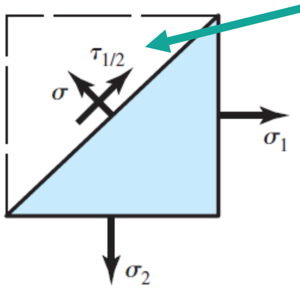
When an element has a particular orientation that all shear-stress components are zero…

- Principal Faces / Directions / Stresses (σ_1, σ_2, σ_3)
  - For plane stress, the stress-free surface contains the 3rd principal stress which is zero.
- Find the 3 principal stresses from the 6 stress components
  - The 3 principal stresses: $\sigma_1, \sigma_2, \sigma_3$ (ordered $\sigma_1 \geq \sigma_2 \geq \sigma_3$)
  - The 6 stress components: $\sigma_x, \sigma_y, \sigma_z, \sigma_{xy}, \sigma_{yz}, \sigma_{zx}$

$$\tau_{\max} = \tau_{1/3}$$

The stress coordinates σ, τ for any arbitrarily located plane will always lie on the boundaries or within the shaded area.





3 principal shear stresses

$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2}$$

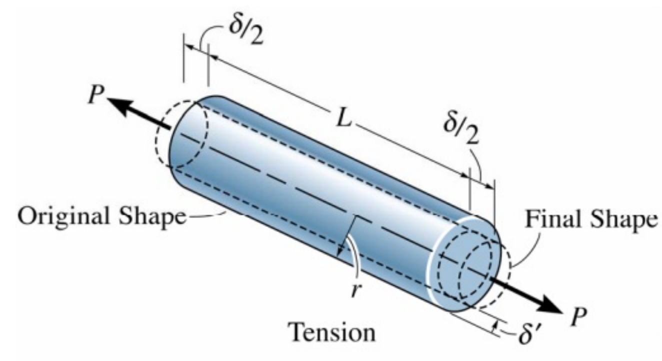
$$\tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2}$$

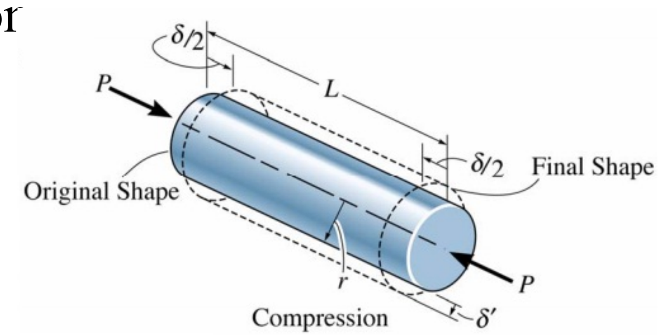
AncoraSIR.com

Elastic Strain

The *Modulus* Between Force and Displacement

- Normal Strain $\sigma = E\epsilon$
- Shear Strain $\tau = G\gamma$
  - the change in a right angle of a stress element when subjected to pure shear stress
- Poisson's Ratio $\epsilon_y = \epsilon_z = -\nu\epsilon_x.$
  - When a material is placed in tension / contraction there exists not only an axial strain, but also negative strain (contraction / tension) perpendicular to the axial strain.





$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

Young's modulus or the modulus of elasticity

$$E = 2G(1 + \nu)$$

shear modulus of elasticity or modulus of rigidity.

AncoraSIR.com

SUSTech Southern University of Science and Technology

Stress Concentration

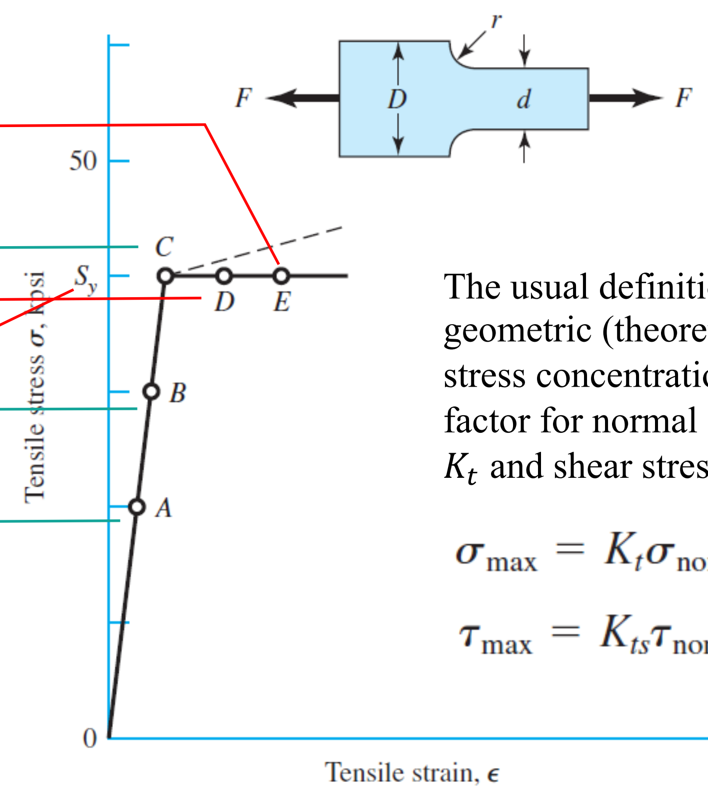
A Highly Localized Effect

- Worst-case Scenario Analysis (Close to "Failure")
  - For an idealized non–strain-strengthening material, the stress-strain curve rises linearly to the yield strength S_y, then proceeds at constant stress, which is equal to S_y.
- Stress-Concentration Factor (SCF)
  - A Ratio between the maximum and nominal stresses

$$K = \sigma_{max}/\sigma_{nom} = S_y/\sigma = 40/40 = 1.$$

$$K = \sigma_{max}/\sigma_{nom} = S_y/\sigma = 40/30 = 1.33.$$

$$K = \sigma_{max}/\sigma_{nom} = 40/20 = 2.$$



The usual definition of geometric (theoretical) stress concentration factor for normal stress K_t and shear stress K_{ts}

$$\sigma_{max} = K_t \sigma_{nom}$$

$$\tau_{max} = K_{ts} \tau_{nom}$$

SUSTech Southern University of Science and Technology

AncoraSIR.com

Failure Theories

☹ No universal theory of failure for the general case of material properties and stress state.

- Hypotheses => Accepted => Theories (for Practices)

- **Ductile materials** (yield criteria) $\varepsilon_f \geq 0.05$
  - Maximum shear stress (MSS)
  - Distortion energy (DE)
  - Ductile Coulomb-Mohr (DCM)

have an identifiable yield strength that is often the same in compression as in tension ($S_{yt} = S_{yc} = S_y$)

- **Brittle materials** (fracture criteria) $\varepsilon_f < 0.05$
  - Maximum normal stress (MNS)
  - Brittle Coulomb-Mohr (BCM)
  - Modified Mohr (MM)

do not exhibit an identifiable yield strength, and are typically classified by ultimate tensile and compressive strengths, S_{ut} and S_{uc}, respectively.

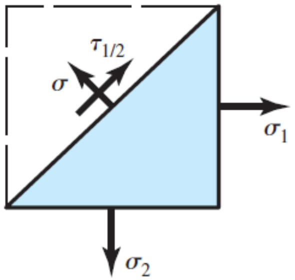
AncoraSIR.com

Maximum-Shear-Stress Theory (MSS)

for Ductile Materials

- *yielding begins whenever the maximum shear stress in any element equals or exceeds the maximum shear stress in a tension-test specimen of the same material when that specimen begins to yield.*
- Postulation that the mechanism of failure is related to the shear fracture at 45˚ line
  - The maximum shear stress occurs on a surface 45˚ from the tensile surface

$$\tau_{max} = \sigma/2$$



$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \geq \boxed{\frac{S_y}{2}}$$

S_{sy}

- Safety Factor n

$$\tau_{max} = \tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\tau_{max} = \frac{S_y}{2n} \qquad \text{or} \qquad \sigma_1 - \sigma_3 = \frac{S_y}{n}$$

Plane Stress is 3D.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

out of plane there is a third principal stress and it is *always zero* for plane stress

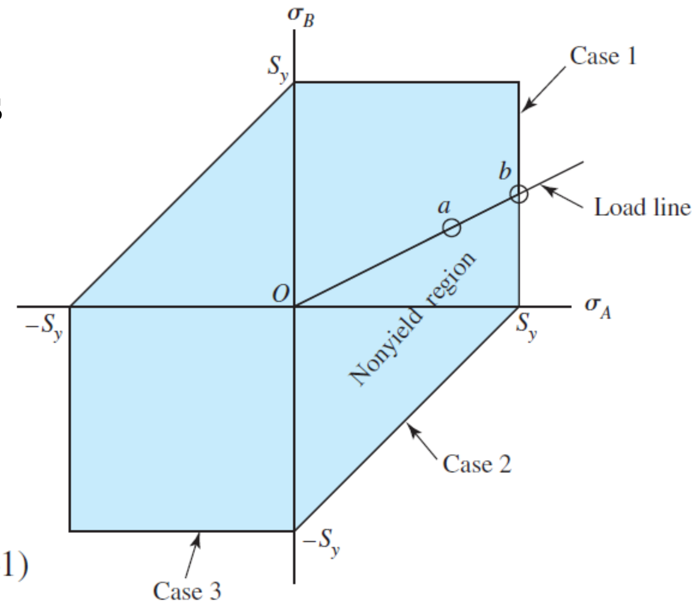
We need a different set of labels ☹

AncoraSIR.com

Cases of MSS Theory

Very Confusing, yet Reasonable

- Label Principal Stress σ_1 as σ_A, and σ_2 as σ_B
- Remind yourself that there is another zero principal stres
- Order them $\sigma_1 \geq \sigma_2 \geq \sigma_3$
- Assume $\sigma_A \geq \sigma_B$



$$\sigma_1 - \sigma_3 \geq S_y \qquad (5\text{–}1)$$

Case 1: $\sigma_A \geq \sigma_B \geq 0$. For this case, $\sigma_1 = \sigma_A$ and $\sigma_3 = 0$. Equation (5–1) reduces to a yield condition of

$$\sigma_A \geq S_y \qquad (5\text{–}4)$$

Case 2: $\sigma_A \geq 0 \geq \sigma_B$. Here, $\sigma_1 = \sigma_A$ and $\sigma_3 = \sigma_B$, and Eq. (5–1) becomes

$$\sigma_A - \sigma_B \geq S_y \qquad (5\text{–}5)$$

Case 3: $0 \geq \sigma_A \geq \sigma_B$. For this case, $\sigma_1 = 0$ and $\sigma_3 = \sigma_B$, and Eq. (5–1) gives

$$\sigma_B \leq -S_y \qquad (5\text{–}6)$$

The factor of safety guarding against yield at point a is given by

- the ratio ($n=Ob/Oa$) of
- Strength (distance to failure at point b) to
- Stress (distance to stress at point a).

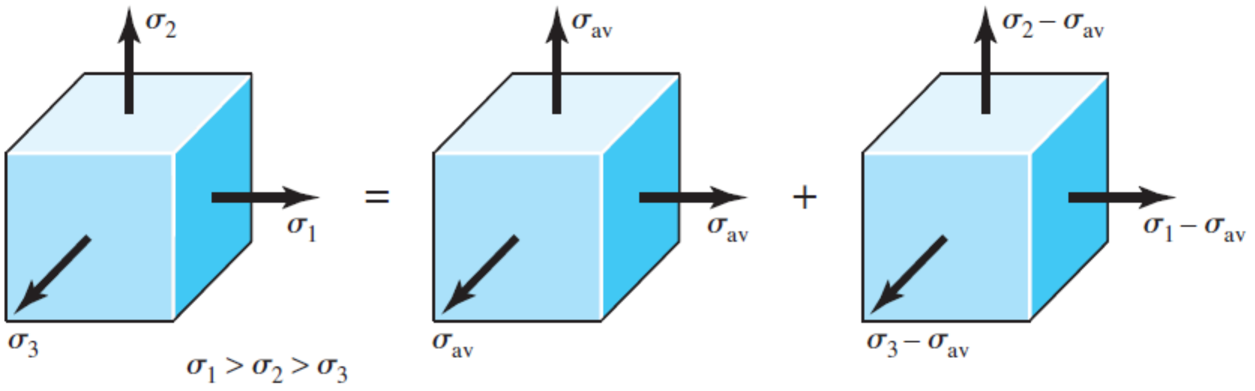
AncoraSIR.com

Distortion-Energy Theory (DE)

for Ductile Materials

- *yielding occurs when the distortion strain energy per unit volume reaches or exceeds the distortion strain energy per unit volume for yield in simple tension or compression of the same material.*
- Postulation that yielding is not a simple tensile or compressive phenomenon at all
  - The observation that ductile materials stressed **hydrostatically** (equal principal stresses) exhibited yield strengths **greatly in excess** of the values given by the simple tension test.

$$\sigma_{av} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \qquad u = u_v + u_d$$



(a) Triaxial stresses (b) Hydrostatic component (c) Distortional component

AncoraSIR.com

von Mises Stress

DE in not as conservative as MMS.

$$u = \frac{1}{2}\epsilon\sigma$$

$$u = \frac{1}{2}[\epsilon_1\sigma_1 + \epsilon_2\sigma_2 + \epsilon_3\sigma_3]$$

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

$$u = \frac{1}{2E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$$

$$\sigma_{1,2,3} = \sigma_{av} = (\sigma_1 + \sigma_2 + \sigma_3)/3$$

$$u_v = \frac{3\sigma_{av}^2}{2E}(1 - 2\nu) = \frac{1 - 2\nu}{6E}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\sigma_1\sigma_2 + 2\sigma_2\sigma_3 + 2\sigma_3\sigma_1)$$

$$u_d = u - u_v = \frac{1 + \nu}{3E}\left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}\right]$$

For a simple tensile test $\sigma_1 = S_y$ and $\sigma_2 = \sigma_3 = 0$,

$$u_d = \frac{1 + \nu}{3E}S_y^2 \rightarrow \left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}\right]^{1/2} \geq S_y \rightarrow \sigma' \geq S_y$$

A single, equivalent, or effective stress, called the *von Mise's stress*

When yield occurs?

$$\sigma \geq S_y.$$

AncoraSIR.com

SUSTech Southern University of Science and Technology

von Mise's Stress in Planar Form

A Special Case when Using Principal Stresses.

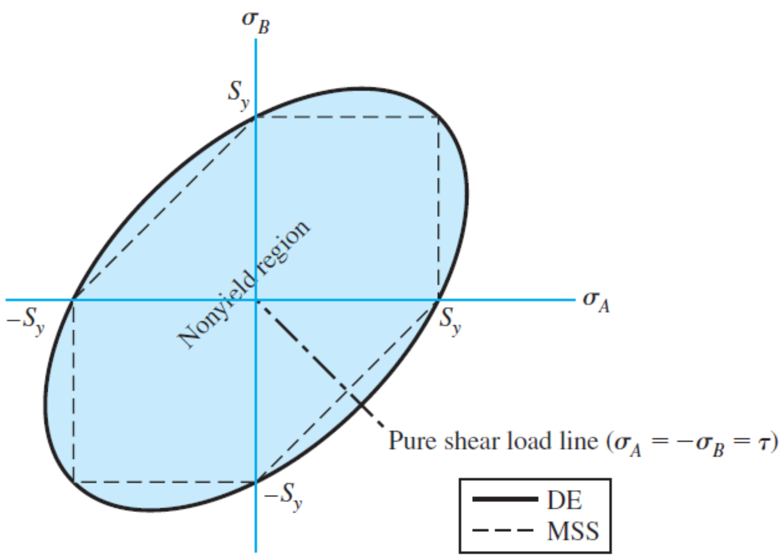
$$\sigma' = \left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}\right]^{1/2}$$

For plane stress, $\sigma_1 = \sigma_A$, $\sigma_2 = \sigma_B$, $\sigma_3 = 0$.

$$\sigma' = (\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2)^{1/2}$$

σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, and τ_{zx}.

$$\sigma' = (\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{1/2}$$



Allow the most complicated stress situation to be represented by *a single quantity*, the ***von Mises stress***, which then can be compared against the yield strength of the material.

Design Equation $\sigma' = \dfrac{S_y}{n}$

Why MSS is more conservative than DE?

- The model for the MSS theory ignores the contribution of the normal stresses on the 45° surfaces of the tensile specimen.

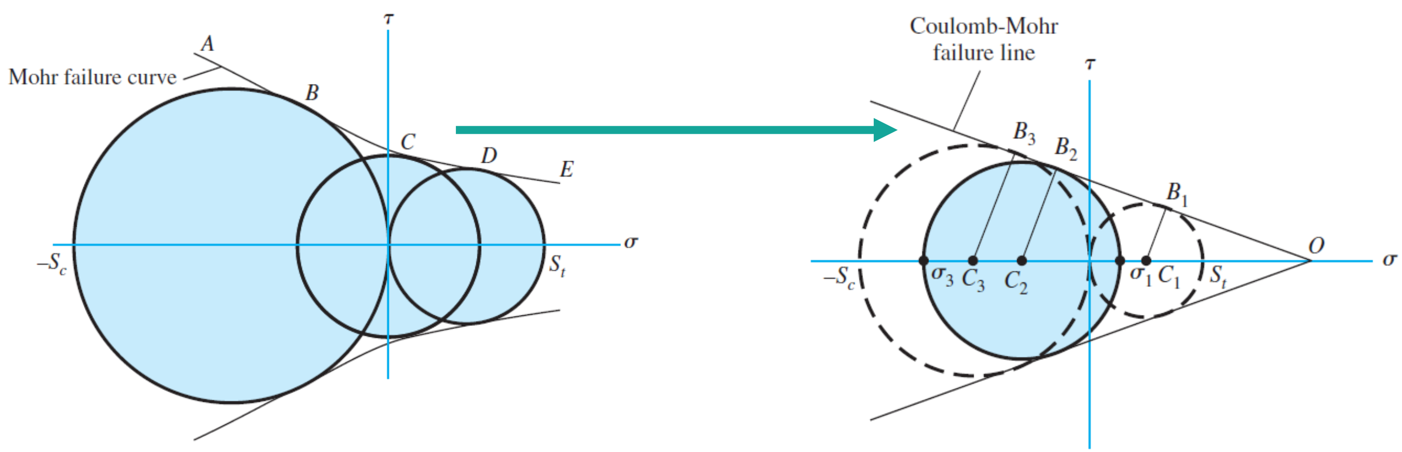
AncoraSIR.com

SUSTech Southern University of Science and Technology

Ductile Coulomb-Mohr Theory (DMT)

Not all materials have compressive strengths equal to their corresponding tensile values.

- Three "simple" tests to yielding if the material can yield, or to rupture.
  - tension, compression, and shear
- Mohr's Failure Envelope
  - use the results of tensile, compressive, and torsional shear tests to construct the three circles, and a curve (ABCDE) tangent to these three circles
  - the stress state in a body growing during loading until one of them became tangent to the failure envelope, thereby defining failure.
- Coulomb-Mohr Theory
  - assumes that the boundary BCD is straight.
  - only need the tensile and compressive strengths.



AncoraSIR.com

SUSTech Southern University of Science and Technology

DCM Failure Envelope for Plane Stress States

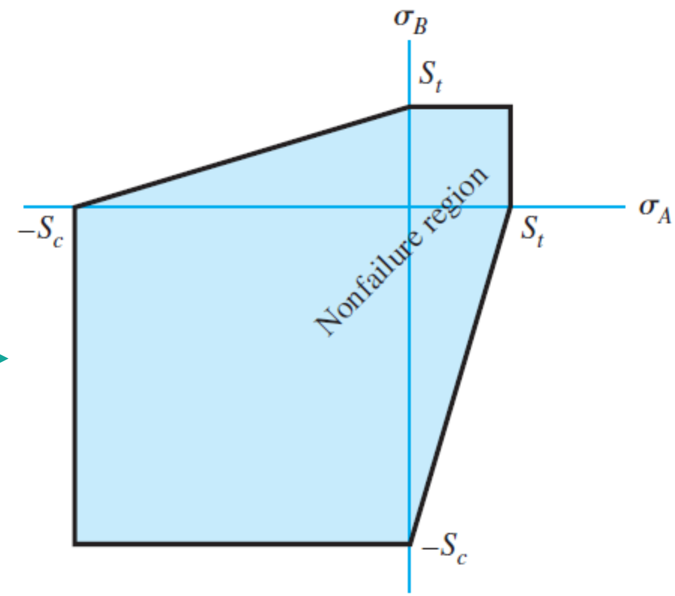
Unified Graphical Representation

- From Geometric Relationship

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1$$

  - Can use either yield strength or ultimate strength

- For Plan Stress



Case 1: $\sigma_A \geq \sigma_B \geq 0$. For this case, $\sigma_1 = \sigma_A$ and $\sigma_3 = 0$. Equation (5–22) reduces to

$$\sigma_A \geq S_t \qquad (5\text{–}23)$$

Case 2: $\sigma_A \geq 0 \geq \sigma_B$. Here, $\sigma_1 = \sigma_A$ and $\sigma_3 = \sigma_B$, and Eq. (5–22) becomes

$$\frac{\sigma_A}{S_t} - \frac{\sigma_B}{S_c} \geq 1 \qquad (5\text{–}24)$$

Case 3: $0 \geq \sigma_A \geq \sigma_B$. For this case, $\sigma_1 = 0$ and $\sigma_3 = \sigma_B$, and Eq. (5–22) gives

$$\sigma_B \leq -S_c \qquad (5\text{–}25)$$

Design Equation

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = \frac{1}{n}$$

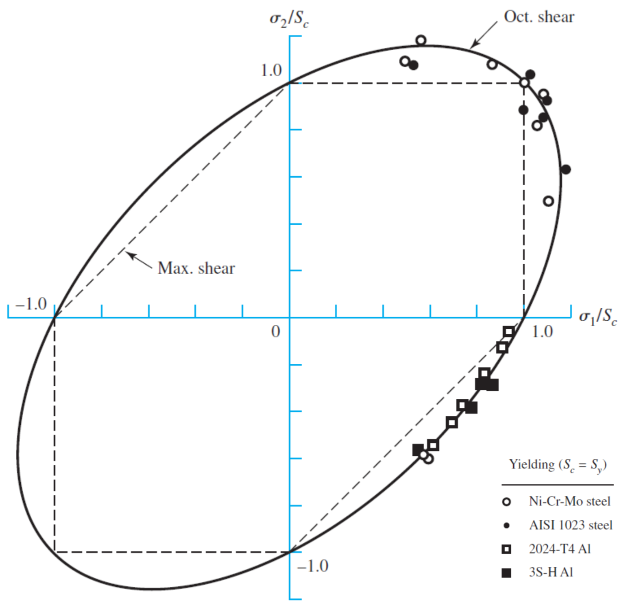
SUSTech Southern University of Science and Technology

AncoraSIR.com

Failure of Ductile Materials

A Summary

- Either the maximum-shear-stress (MSS) theory or the distortion-energy (DE) theory is acceptable for design and analysis of materials that would fail in a ductile manner
  - For design purposes, the maximum-shear-stress (MSS) theory is easy, quick to use, and conservative.
  - If the problem is to learn why a part failed, then the distortion-energy (DE) theory may be the best to use.
  - For ductile materials with unequal yield strengths, the Mohr or Coulomb-Mohr theory (DCM) is the best available.
- For design purposes, a larger factor of safety may be warranted when using such a failure theory
  - Though a failure curve passing through the center of the experimental data is typical of the data, its reliability from a statistical standpoint is about 50%.



AncoraSIR.com

Maximum-Normal-Stress Theory

for Brittle Materials

- Failure occurs whenever one of the three principal stresses equals or exceeds the strength

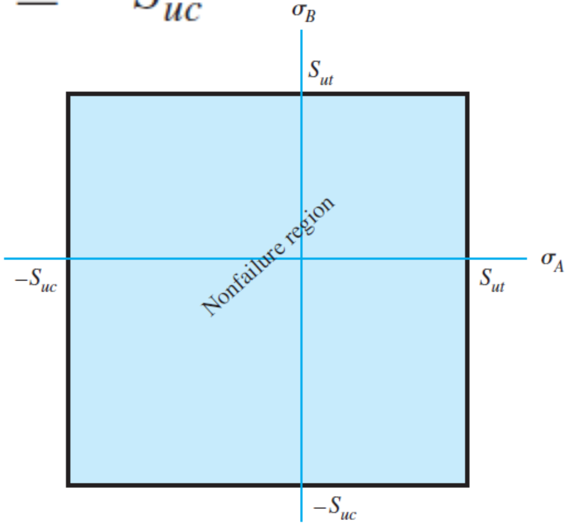
$$\sigma_1 \geq S_{ut} \qquad \text{or} \qquad \sigma_3 \leq -S_{uc}$$

ultimate tensile compressive strengths

- For Plan Stress $\sigma_A \geq S_{ut}$ or $\sigma_B \leq -S_{uc}$
- Design Equation

$$\sigma_A = \frac{S_{ut}}{n} \qquad \text{or} \qquad \sigma_B = -\frac{S_{uc}}{n}$$

- Problems in the 4th quadrant.



AncoraSIR.com

Modifications of the Mohr Theory

Brittle-Coulomb-Mohr (BCM) theory & modified Mohr (MM) theory

- Be restricted to plane stress and be of the design type incorporating the factor of safety.
- Brittle-Coulomb-Mohr (BCM)

$$\sigma_A = \frac{S_{ut}}{n} \qquad \sigma_A \geq \sigma_B \geq 0$$

$$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \qquad \sigma_A \geq 0 \geq \sigma_B$$

$$\sigma_B = -\frac{S_{uc}}{n} \qquad 0 \geq \sigma_A \geq \sigma_B$$

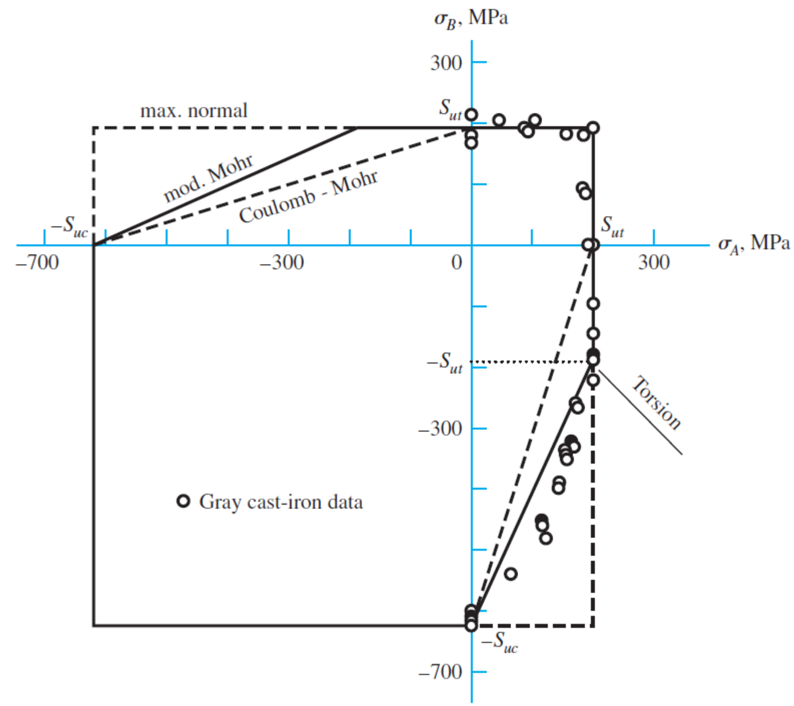
- Modified Mohr (MM)

$$\sigma_A = \frac{S_{ut}}{n} \qquad \sigma_A \geq \sigma_B \geq 0$$

$$\sigma_A \geq 0 \geq \sigma_B \quad \text{and} \quad \left|\frac{\sigma_B}{\sigma_A}\right| \leq 1$$

$$\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \qquad \sigma_A \geq 0 \geq \sigma_B \quad \text{and} \quad \left|\frac{\sigma_B}{\sigma_A}\right| > 1$$

$$\sigma_B = -\frac{S_{uc}}{n} \qquad 0 \geq \sigma_A \geq \sigma_B$$

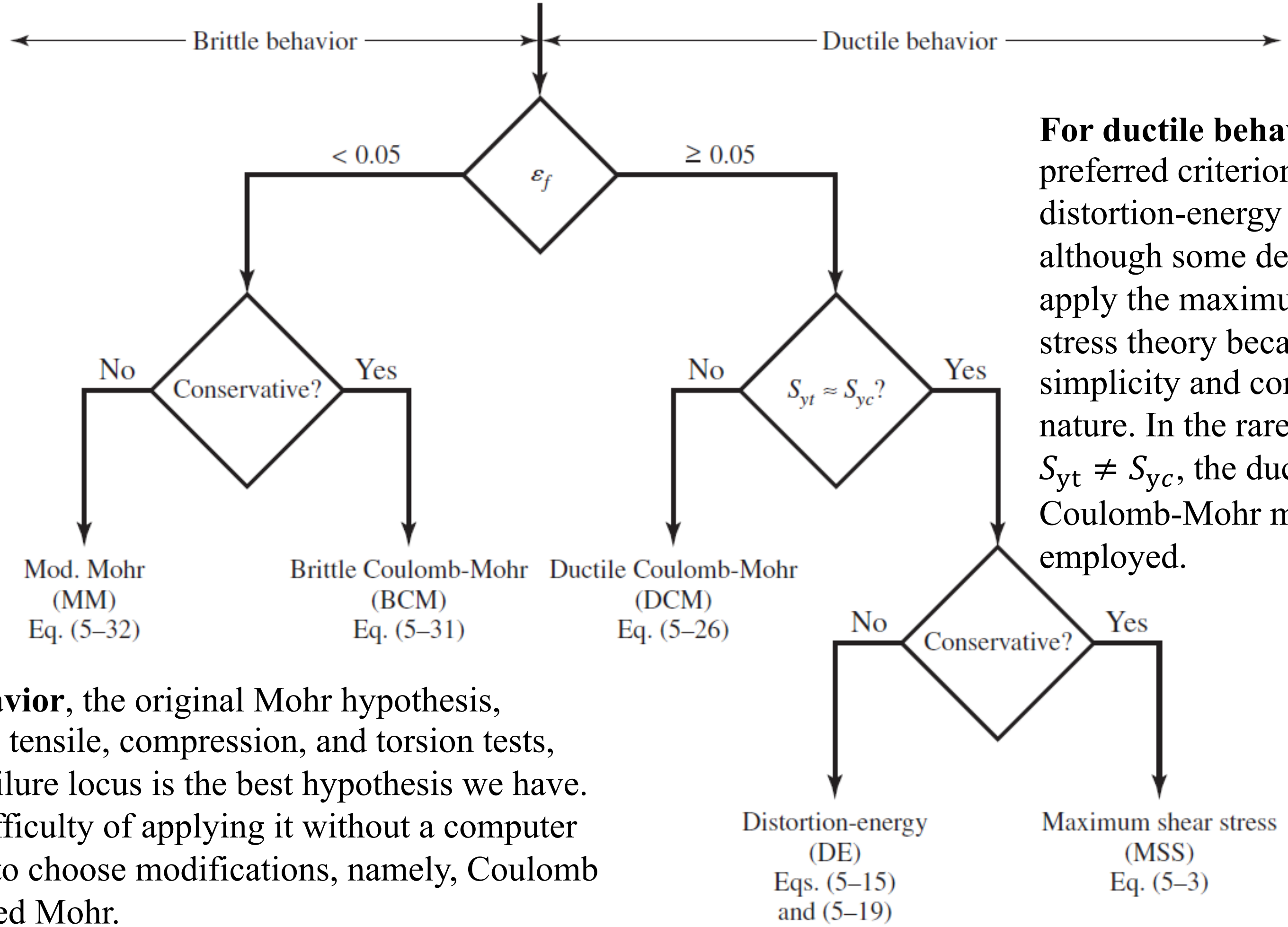


- In the 1st quadrant the data appear on both sides and along the failure curves of maximum-normal-stress, Coulomb-Mohr, and modified Mohr. All failure curves are the same, and data fit well.
- In the 4th quadrant the modified Mohr theory represents the data best, whereas the maximum-normal-stress theory does not.

AncoraSIR.com

SUSTech
Southern University of Science and Technology

Selection of Failure Criteria



For ductile behavior, the preferred criterion is the distortion-energy theory, although some designers also apply the maximum-shear-stress theory because of its simplicity and conservative nature. In the rare case when $S_{yt} \neq S_{yc}$, the ductile Coulomb-Mohr method is employed.

For brittle behavior, the original Mohr hypothesis, constructed with tensile, compression, and torsion tests, with a curved failure locus is the best hypothesis we have. However, the difficulty of applying it without a computer leads engineers to choose modifications, namely, Coulomb Mohr, or modified Mohr.

AncoraSIR.com

SUSTech Southern University of Science and Technology

Introduction to Fracture Mechanics

Focus on crack growth until it becomes critical, then remove it from service.

- Linear elastic fracture mechanics (LEFM)
  - Inspection and maintenance are essential in the decision to retire parts before cracks reach catastrophic size.
  - Where human safety is concerned, periodic inspections for cracks are mandated by codes and government ordinance.

- Relatively Brittle
  - (rigorously defined in the test procedures)
  - Fracture without yielding occurring throughout the fractured cross section
  - Glass, hard steels, strong aluminum alloys, and even low-carbon steel below the ductile-to-brittle transition temperature can be analyzed in this way

SUSTech
Southern University of Science and Technology

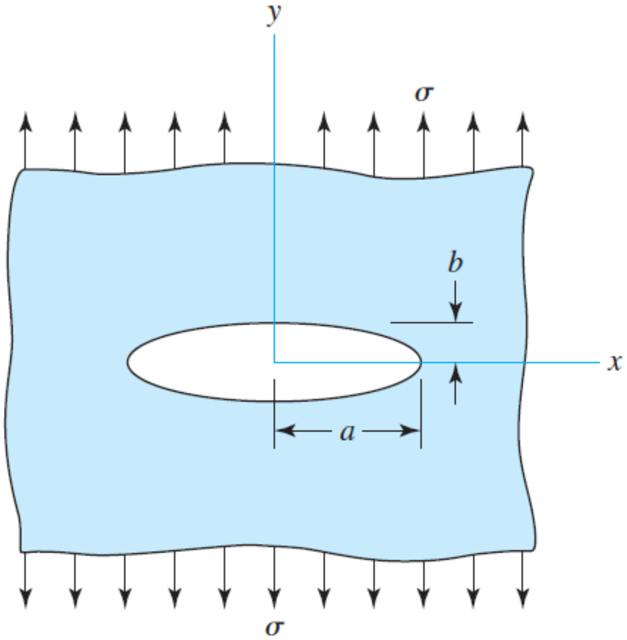
AncoraSIR.com

Quasi-Static Fracture

brittle fracture

maximum stress occurs at $(\pm a, 0)$

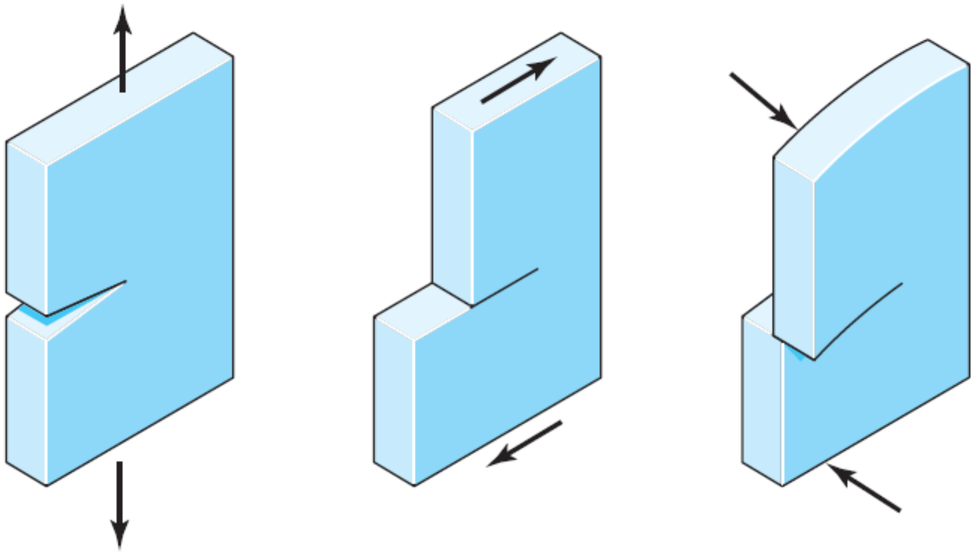
$$(\sigma_y)_{\max} = \left(1 + 2\frac{a}{b}\right)\sigma$$



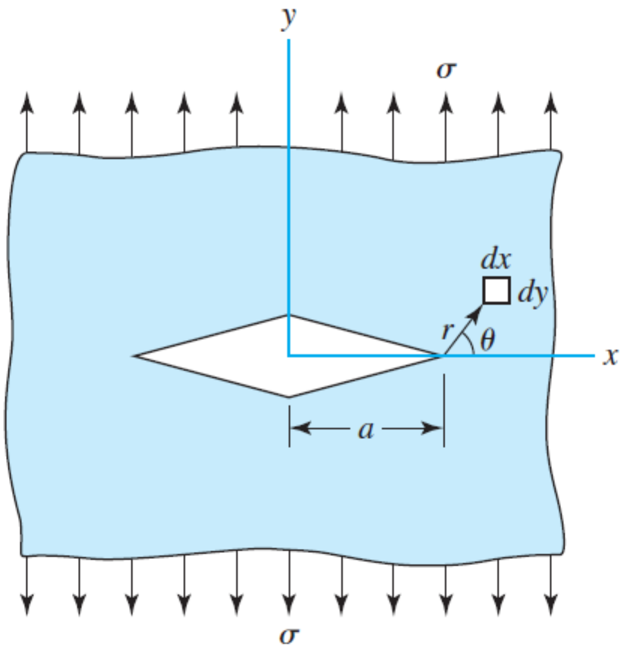
- For a fine crack,

$$b/a \to 0,\ (\sigma_y)_{\max} \to \infty.$$

Crack Modes and the Stress Intensity Factor



(*a*) Mode I (*b*) Mode II (*c*) Mode III



opening crack propagation mode, the most common and important mode

sliding mode

tearing mode

$K_I = \sigma\sqrt{\pi a}$ stress intensity factor

$K_I = \beta\sigma\sqrt{\pi a}$ stress intensity modification factor

AncoraSIR.com

SUSTech
Southern University of Science and Technology

Next class

- **Discussion for Group 2**: Mechanism Design
- Friday 0800-1000, Sep 27
- Room 202, 1 Lychee Park

- **Lab for Group 1**: Mechanism Design
- Friday 0800-1000, Sep 27
- Room 412, 5 Wisdom Valley

Thank you!

Song Chaoyang (songcy@sustech.edu.cn)

- Xiao Xiaochuan (xiaoxc@sustech.edu.cn)
- Yu Chengming (11930324@mail.sustech.edu.cn)
- Zhu Wenpei (11930368@mail.sustech.edu.cn)
- Guo Ning (11930729@mail.sustech.edu.cn)

AncoraSIR.com

SUSTech Southern University of Science and Technology