ME303 Introduction to Mechanical Design

Lecture 15
Flexible Mechanical Elements

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Agenda

Week 14, Wednesday, Dec 11, 2019

- Belts
- Flat- and Round-Belt Drives
- V Belts
- Timing Belts
- Roller Chain
- Wire Rope
- Flexible Shafts

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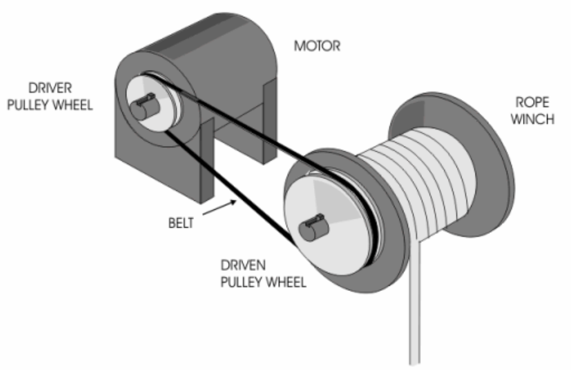
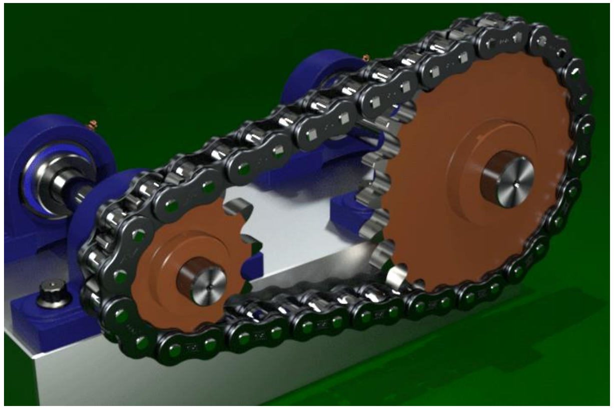
Flexible Mechanical Elements

Used in conveying systems and in the transmission of power over comparatively long distances.

- As a replacement for gears, shafts, bearings, and other relatively rigid power-transmission devices.
  - For power transmission
  - To increase or decrease speed or torque
  - Mainly used in conveying systems





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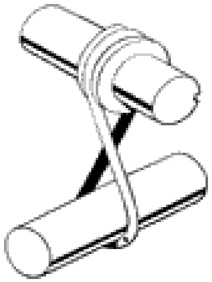
Flexible Mechanical Elements

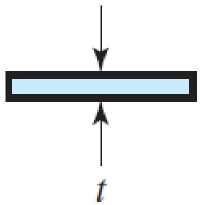
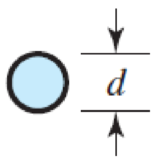
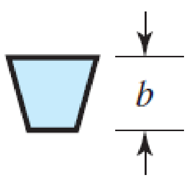
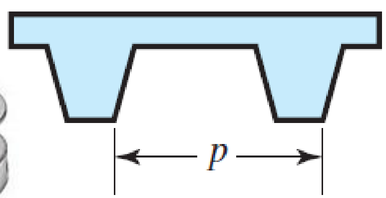
Advantages



- Simplifies the design of a machine
- Substantially reduces the cost
- Shock absorption, damping out or isolate the vibration effects
- Generally high efficiency that ranges from 90% to 98%
- Can drive several shafts from a single power source
- Can tolerate some degree of misalignment

- Have a finite life, so replacement is required after the first sign of deterioration.

Four Principal Types of Belts

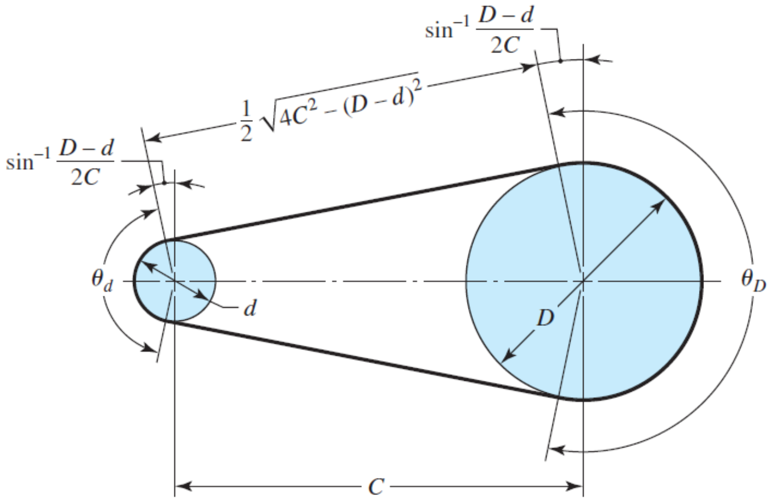
Belt Type	Figure	Joint	Size Range	Center Distance
Flat 		Yes	$t = \begin{cases} 0.03 \text{ to } 0.20 \text{ in} \\ 0.75 \text{ to } 5 \text{ mm} \end{cases}$	No upper limit
Round 		Yes	$d = \frac{1}{8} \text{ to } \frac{3}{4} \text{ in}$	No upper limit
V		None	$b = \begin{cases} 0.31 \text{ to } 0.91 \text{ in} \\ 8 \text{ to } 19 \text{ mm} \end{cases}$	Limited
Timing 		None	$p = 2$ mm and up	Limited

Characteristics of Belts

- In all cases, the pulley axes must be separated by a certain minimum distance, depending upon the belt type and size, to operate properly.
- They may be used for long center distances.
- Except for timing belts, there is some slip and creep, and so the angular-velocity ratio between the driving and driven shafts is neither constant nor exactly equal to the ratio of the pulley diameters.
- In some cases an idler or tension pulley can be used to avoid adjustments in center distance that are ordinarily necessitated by age or the installation of new belts.

Flat-belt Geometry

Open belt

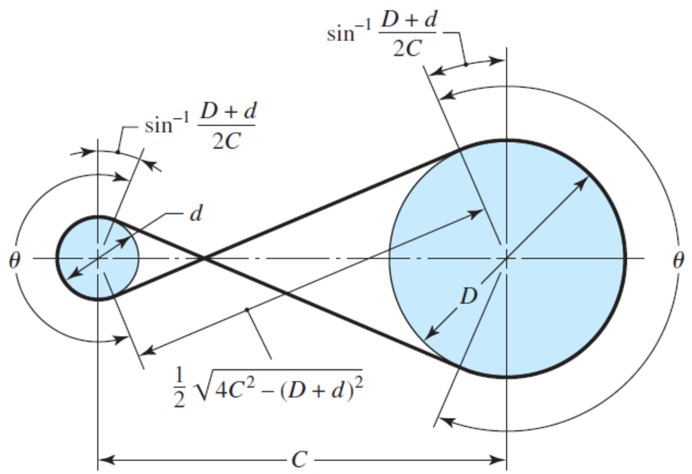


$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C}$$

$$L = \sqrt{4C^2-(D-d)^2} + \frac{1}{2}(D\theta_D + d\theta_d)$$

Crossed belt



$$\theta = \pi + 2\sin^{-1}\frac{D+d}{2C}$$

$$L = \sqrt{4C^2-(D+d)^2} + \frac{1}{2}(D+d)\theta$$

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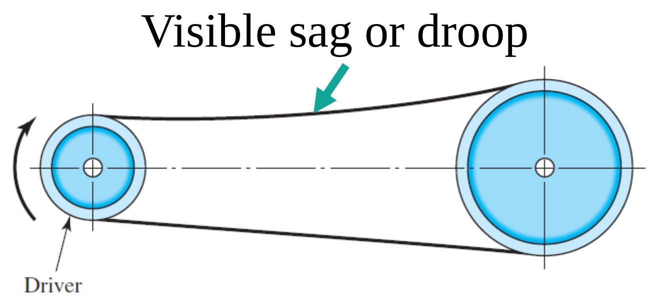
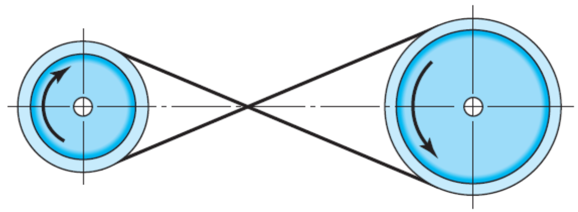
Non-reversing and Reversing Belt Drives

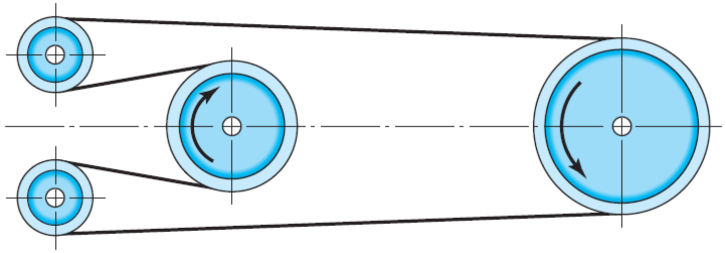
- Non-reversing open belt.



- Reversing crossed belt.
  - Crossed belts must be separated to prevent rubbing if high-friction materials are used.



- Reversing open-belt drive.



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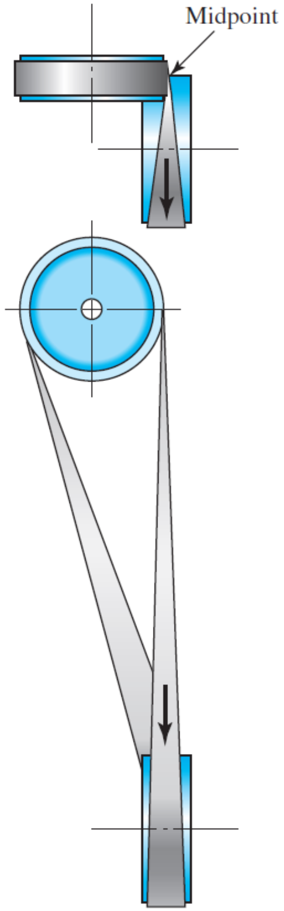
Quarter-twist belt drive

Other Belt Drive Arrangements



- A flat-belt drive with out-of-plane pulleys
  - An idler guide pulley must be used if motion is to be in both directions.
  - The shafts need not to be at right angles.
  - The pulleys must be positioned so that the belt leaves each pulley in the midplane of the other pulley face.
- Other arrangements may require guide pulleys to achieve this condition.

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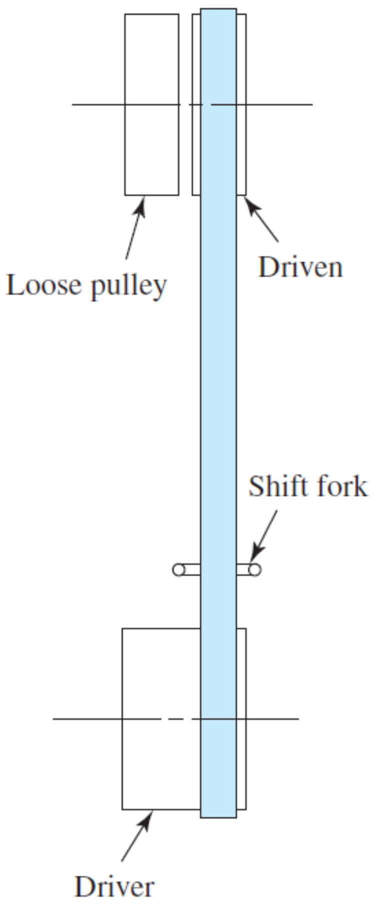
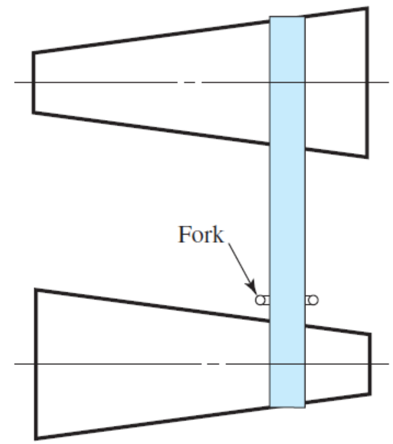
As a Clutch or For Speed Variation

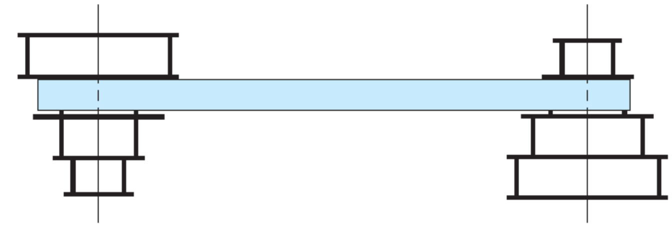
Other Belt Drive Arrangements



- This drive eliminates the need for a clutch.
  - Flat belt can be shifted left or right by use of a fork.
  - Between a loose and a right pulley
- Variable-speed belt drives.



Commonly used only for flat belts.



Can also be used for V belts and round belts by using grooved sheaves

About Flat Belts

- Materials
  - Made of urethane and also of rubber-impregnated fabric reinforced with steel wire or nylon cords to take the tension load.
  - One or both surfaces may have a friction surface coating.
- Characteristics
  - Quiet
  - Efficient at high speeds
  - Can transmit large amounts of power over long center distances.
- Common use
  - Usually purchased by the roll
  - Then cut and the ends are joined by using special kits furnished by the manufacturer.
  - Two or more flat belts running side by side, instead of a single wide belt, are often used to form a conveying system.

About V Belt

- Material
  - Fabric and cord, usually cotton, rayon, or nylon, and impregnated with rubber.
- Characteristics
  - Slightly less efficient than flat belts
  - But a number of them can be used on a single sheave, thus making a multiple drive.
- Common Use
  - In contrast with flat belts, V belts are used with similar sheaves and at shorter center distances.
  - V belts are made only in certain lengths and have no joints.

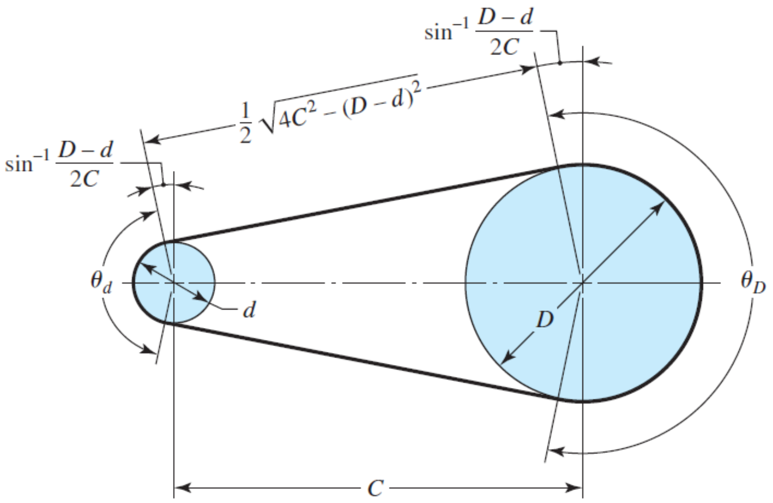
About Timing Belts

- Materials
  - Rubberized fabric and steel wire
  - Have teeth that fit into grooves cut on the periphery of the sprockets.
- Characteristics
  - The timing belt does not stretch or slip
  - Consequently transmits power at a constant angular-velocity ratio.
- Advantages
  - No initial tension is necessary, so that fixed-center drives may be used.
  - The elimination of the restriction on speeds.
  - The teeth make it possible to run at nearly any speed, slow or fast.
- Disadvantages
  - The first cost of the belt,
  - The necessity of grooving the sprockets
  - The attendant dynamic fluctuations caused at the belt-tooth meshing frequency.

Flat- and Round-Belt Drives

Modern flat-belt drives consist of a strong elastic core surrounded by an elastomer.

- Has an efficiency of about 98 percent, like a gear drive
- Produce very little noise
- Absorb more torsional vibration from the system



$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C}$$

D = diameter of large pulley
d = diameter of small pulley
C = center distance
θ = angle of contact

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Flat-belt-drive Theory

Firbank's Explanation

- Caused by *elastic creep* and associated with *sliding friction*
  - A change in belt tension due to friction forces between the belt and pulley will cause the belt to elongate or contract and move relative to the surface of the pulley.
- The angle of contact at the power transmitting portion
  - The action at the driving pulley is such that the belt moves more slowly than the surface speed of the pulley
  - the *effective arc* (power transmission portion) and the *idle arc*
- Substantially more power is transmitted by *static friction* than sliding friction.
- The coefficient of friction for a belt
  - With a nylon core and leather surface was typically 0.7,
  - but that it could be raised to 0.9 by employing special surface finishes.

A Different Model

Assume the friction force on the belt is proportional to the normal pressure along the arc of contact

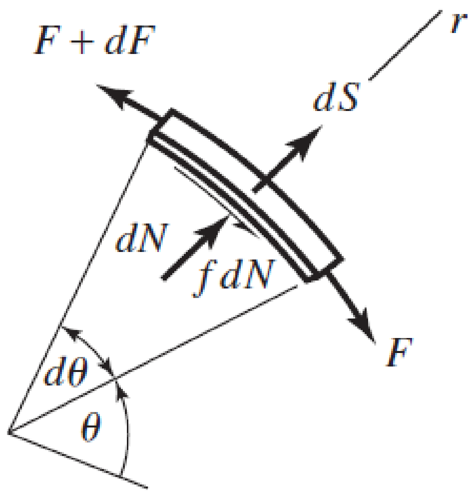
- Free body of an infinitesimal element of a flat belt in contact with a pulley
  - The differential force *dS* is due to centrifugal force,
  - *dN* is the normal force between the belt and pulley,
  - *fdN* is the shearing traction due to friction at the point of slip.
  - The belt width is *b* and the thickness is *t*.

$$dS = (mr\,d\theta)r\omega^2 = mr^2\omega^2\,d\theta = mV^2\,d\theta = F_c\,d\theta$$

$$\sum F_r = -(F + dF)\frac{d\theta}{2} - F\frac{d\theta}{2} + dN + dS = 0 \rightarrow dN = F\,d\theta - dS$$

$$\sum F_t = -f\,dN - F + (F + dF) = 0$$

$$dF = f\,dN = f\,F\,d\theta - f\,dS = f\,F\,d\theta - f\,mr^2\omega^2\,d\theta \rightarrow \frac{dF}{d\theta} - f\,F = -f\,mr^2\omega^2$$



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Mechanic Analysis

tight-side tension F_1

loose-side tension F_2

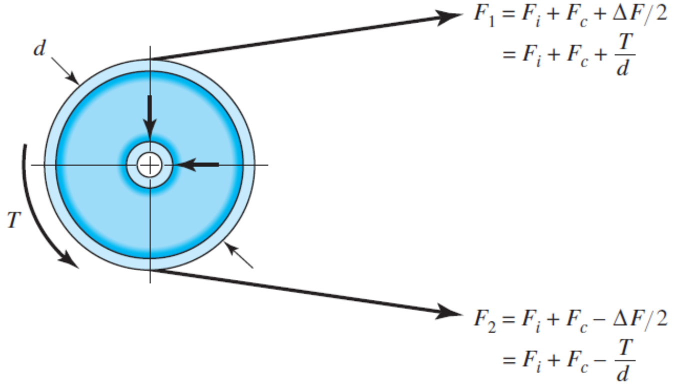
$$\frac{dF}{d\theta} - fF = -fmr^2\omega^2 \qquad F = A\exp(f\theta) + mr^2\omega^2$$

Assuming θ starts at the loose side, the boundary condition that F at θ=0 equals F_2 gives A = F_2 - mr^2v^2.

$$F = (F_2 - mr^2\omega^2)\exp(f\theta) + mr^2\omega^2$$

At the end of the angle of wrap ϕ, the tight side,

$$F|_{\theta=\phi} = F_1 = (F_2 - mr^2\omega^2)\exp(f\phi) + mr^2\omega^2$$



the belting equation

$$\frac{F_1 - mr^2\omega^2}{F_2 - mr^2\omega^2} = \frac{F_1 - F_c}{F_2 - F_c} = \exp(f\phi)$$

$$F_c = mr^2\omega^2$$

$$F_1 - F_2 = (F_1 - F_c)\frac{\exp(f\phi) - 1}{\exp(f\phi)}$$

$$F_1 - F_2 = \frac{2T}{d}$$

F_i = initial tension

F_c = hoop tension due to centrifugal force

$\Delta F/2$ = tension due to the transmitted torque T

d = diameter of the pulley

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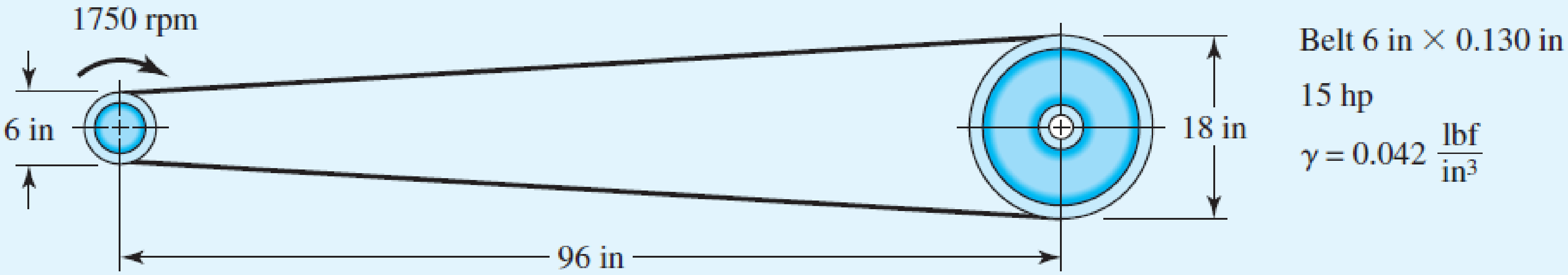
Example of Flat-Belt Analysis

A polyamide A-3 flat belt 6 in wide is used to transmit 15 hp under light shock conditions where $K_s = 1.25$, and a factor of safety equal to or greater than 1.1 is appropriate. The pulley rotational axes are parallel and in the horizontal plane. The shafts are 8 ft apart. The 6-in driving pulley rotates at 1750 rev/min in such a way that the loose side is on top. The driven pulley is 18 in in diameter. See Fig. 17–10. The factor of safety is for unquantifiable exigencies.

(*a*) Estimate the centrifugal tension F_c and the torque T.

(*b*) Estimate the allowable F_1, F_2, F_i and allowable power H_a.

(*c*) Estimate the factor of safety. Is it satisfactory?



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Estimate the centrifugal tension F_c and the torque T

(*a*) Eq. (17–1): $$\phi = \theta_d = \pi - 2\sin^{-1}\left[\frac{18 - 6}{2(8)12}\right] = 3.0165 \text{ rad}$$

Table 17–2: $$\gamma = 0.042 \text{ lbf/in}^3 \qquad f = 0.8 \qquad F_a = 100 \text{ lbf/in}$$

$$\exp(f\phi) = \exp[0.8(3.0165)] = 11.17$$

$$V = \pi(6)1750/12 = 2749 \text{ ft/min}$$

$$w = 12\gamma bt = 12(0.042)6(0.130) = 0.393 \text{ lbf/ft}$$

Eq. (*e*): $$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{0.393}{32.17}\left(\frac{2749}{60}\right)^2 = 25.6 \text{ lbf}$$

$$T = \frac{63\,025 H_{\text{nom}} K_s n_d}{n} = \frac{63\,025(15)1.25(1.1)}{1750}$$

$$= 742.8 \text{ lbf} \cdot \text{in}$$

Estimate the allowable F_1, F_2, F_i

(*b*) The necessary $(F_1)_a - F_2$ to transmit the torque T, from Eq. (*h*), is

$$(F_1)_a - F_2 = \frac{2T}{d} = \frac{2(742.8)}{6} = 247.6 \text{ lbf}$$

For polyamide belts $C_v = 1$, and from Table 17–4 $C_p = 0.70$. From Eq. (17–12) the allowable largest belt tension $(F_1)_a$ is

$$(F_1)_a = bF_aC_pC_v = 6(100)0.70(1) = 420 \text{ lbf}$$

$$F_2 = (F_1)_a - [(F_1)_a - F_2] = 420 - 247.6 = 172.4 \text{ lbf}$$

and from Eq. (*i*)

$$F_i = \frac{(F_1)_a + F_2}{2} - F_c = \frac{420 + 172.4}{2} - 25.6 = 270.6 \text{ lbf}$$

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Estimate the factor of safety. Is it satisfactory?

The combination $(F_1)_a$, F_2, and F_i will transmit the design power of $H_a = H_{\text{nom}} K_s n_d = 15(1.25)(1.1) = 20.6$ hp and protect the belt. We check the friction development by solving Eq. (17–7) for f':

$$f' = \frac{1}{\phi} \ln \frac{(F_1)_a - F_c}{F_2 - F_c} = \frac{1}{3.0165} \ln \frac{420 - 25.6}{172.4 - 25.6} = 0.328$$

As determined earlier, $f = 0.8$. Since $f' < f$, there is no danger of slipping.

(*c*) From step 9 on p. 880,

$$n_{fs} = \frac{H_a}{H_{\text{nom}} K_s} = \frac{20.6}{15(1.25)} = 1.1 \qquad \text{(as expected)}$$

The belt is satisfactory and the maximum allowable belt tension exists. If the initial tension is maintained, the capacity is the design power of 20.6 hp.

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Initial Tension

The key to the functioning of the flat belt as intended

- Weighted idler pulley
  - Motorized pulley drive
- Pivoted motor mount
- Catenary-induced tension
  - the weight of the belt itself can provide the initial tension.

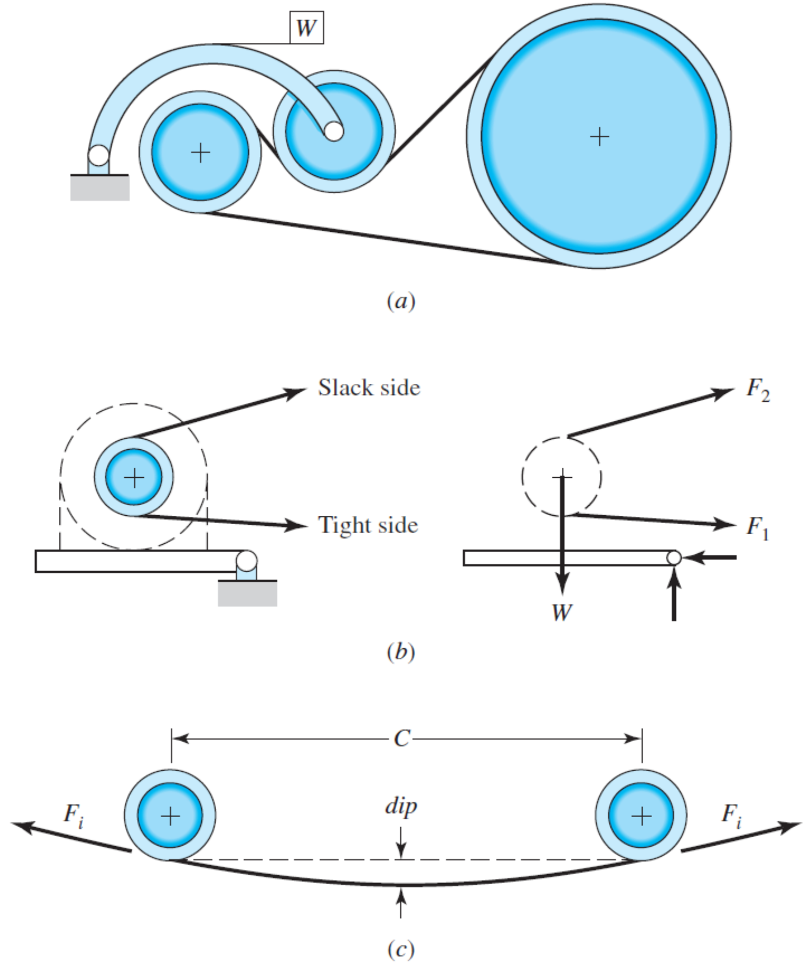
$$dip = \frac{12(C/12)^2 w}{8F_i} = \frac{C^2 w}{96F_i}$$

dip = dip, in

C = center-to-center distance, in

w = weight per foot of the belt, lbf/ft

F_i = initial tension, lbf



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A Decision Set for a Flat Belt

- Function: power, speed, durability, reduction, service factor, *C*
- Design factor: n_d
- Initial tension maintenance
- Belt material
- Drive geometry, *d*, *D*
- Belt thickness: *t*
- Belt width: *b*

Depending on the problem, some or all of *the last four* could be design variables.

Belt cross-sectional area is really the design decision, but available belt thicknesses and widths are discrete choices.

Available dimensions are found in suppliers' catalogs.

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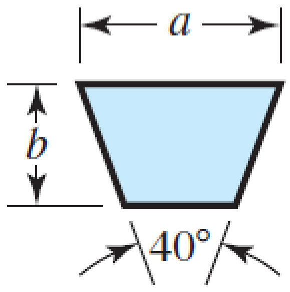
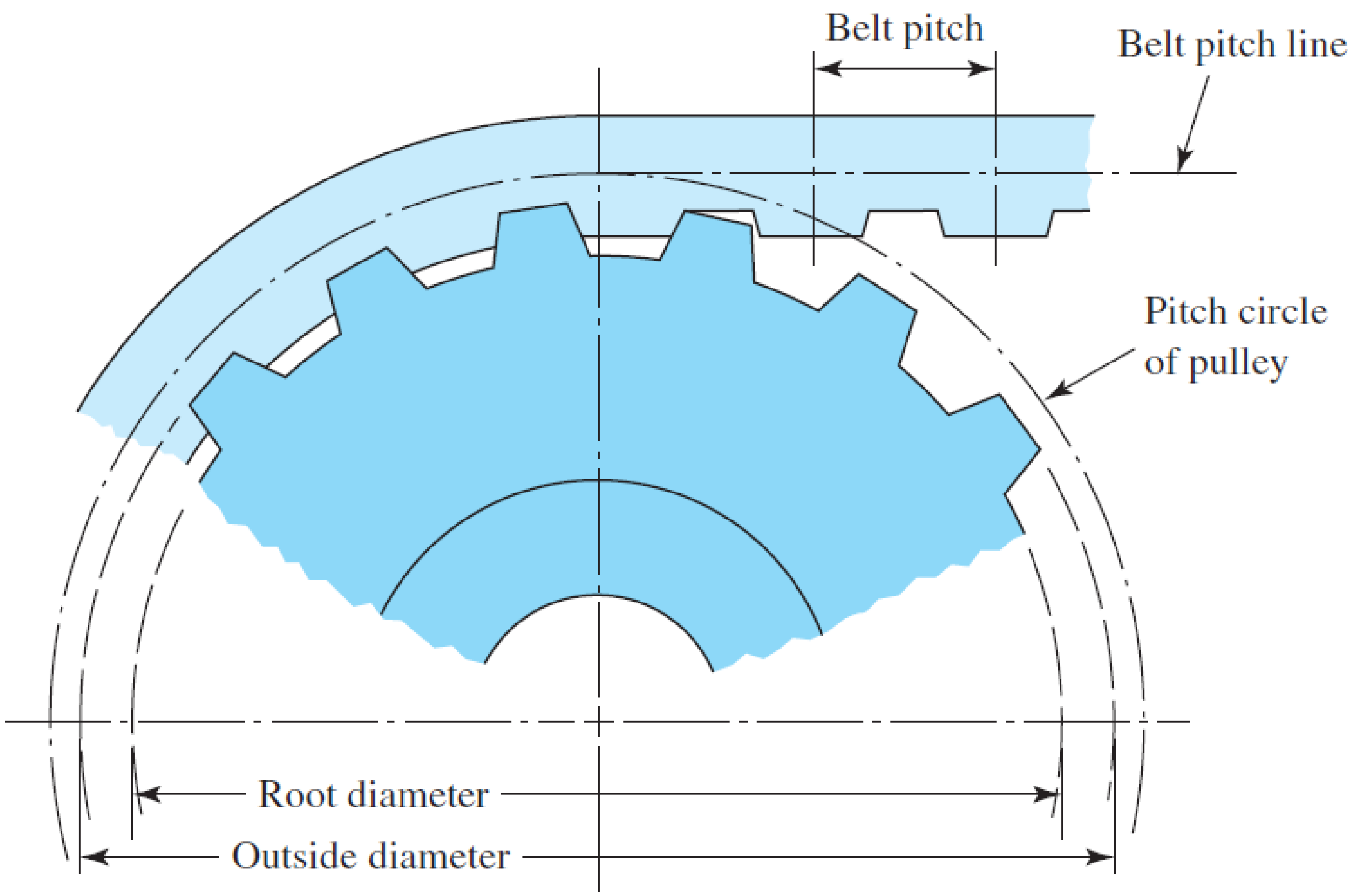
V Belts

- The cross-sectional dimensions of V belts have been standardized by manufacturers, with each section designated by a letter of the alphabet for sizes in inch dimensions.



Belt Section	Width a, in	Thickness b, in	Minimum Sheave Diameter, in	hp Range, One or More Belts
A	$\frac{1}{2}$	$\frac{11}{32}$	3.0	$\frac{1}{4}$–10
B	$\frac{21}{32}$	$\frac{7}{16}$	5.4	1–25
C	$\frac{7}{8}$	$\frac{17}{32}$	9.0	15–100
D	$1\frac{1}{4}$	$\frac{3}{4}$	13.0	50–250
E	$1\frac{1}{2}$	1	21.6	100 and up

Timing Belts



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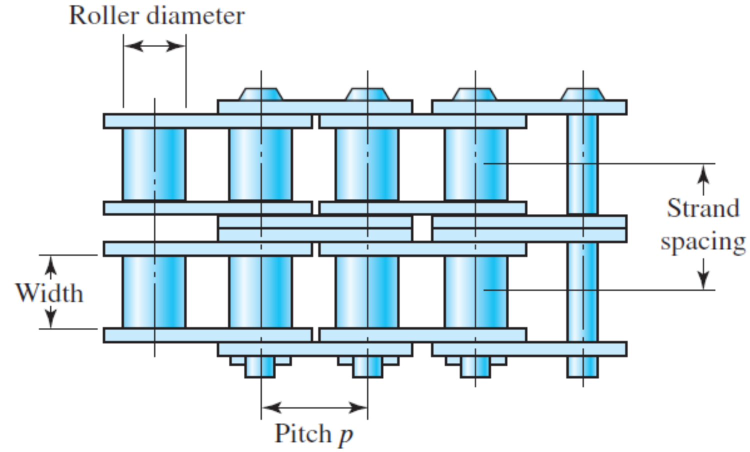
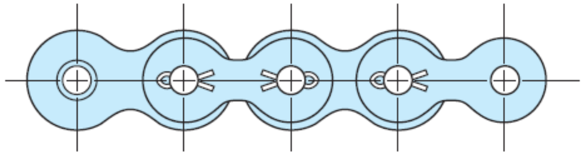
Characteristics of Timing Belts

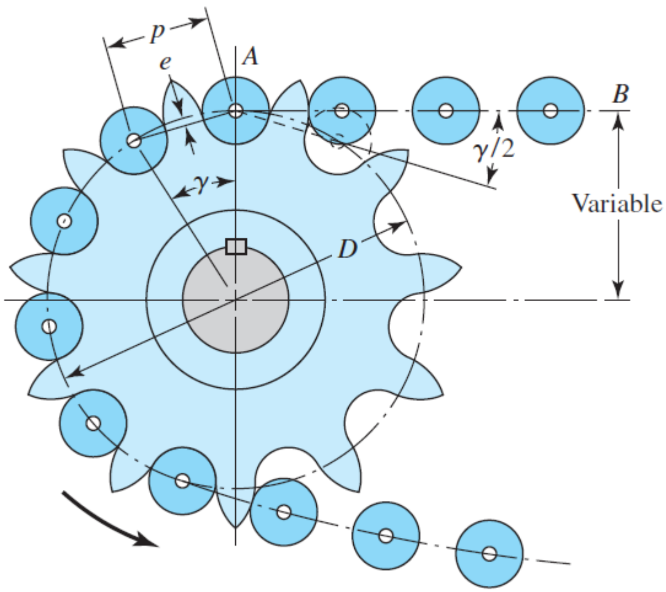
- Made of a rubberized fabric coated with a nylon fabric, and has steel wire within to take the tension load.
- Has teeth that fit into grooves cut on the periphery of the pulleys.
- *Does not stretch appreciably or slip* and consequently transmits power at a constant angular-velocity ratio.
- *No initial tension is needed.*
- Such belts can operate over a very *wide range of speeds*, have efficiencies in the range of *97 to 99 percent*, require *no lubrication*, and are *quieter* than chain drives.
- There is no chordal-speed variation, as in chain drives, and so they are an attractive solution *for precision-drive requirements*.

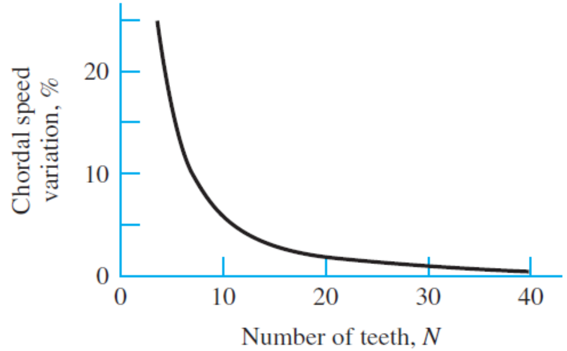
Roller Chain

- Basic features
  - a constant ratio, since no slippage or creep is involved;
  - long life
  - the ability to drive a number of shafts from a single source of power.









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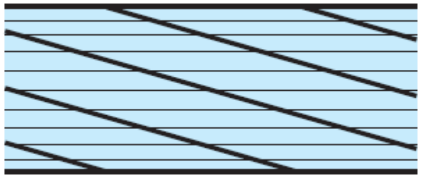
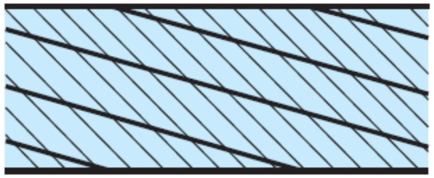
Wire Rope

- The regular lay
  - Has the wire twisted in one direction to form the strands, and the strands twisted in the opposite direction to form the rope.
  - In the completed rope the visible wires are approximately parallel to the axis of the rope.
  - Regular-lay ropes do not kink or untwist and are easy to handle.

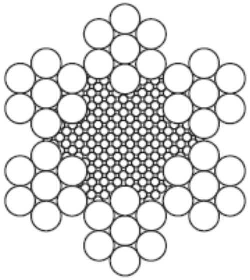
- Lang-lay ropes
  - Have the wires in the strand and the strands in the rope twisted in the same direction, and hence the outer wires run diagonally across the axis of the rope.
  - More resistant to abrasive wear and failure due to fatigue than are regular-lay ropes, but they are more likely to kink and untwist.



(*a*) Regular lay



(*b*) Lang lay

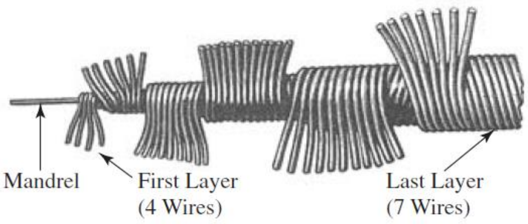


(*c*) Section of 6 × 7 rope

Flexible Shafts

Transmit motion or power around corners

- By winding several layers of wire around a central core
- *The power-drive shaft* for the transmission of power in a single direction
  - Rotation should be in a direction such that the outer layer is wound up
- *The remote-control or manual-control shaft* for the transmission of motion in either direction.
  - The torsional deflection is approximately the same for either direction of rotation



(*a*)

(*b*)

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Next class

- **Lab for Group 1**: Design Consultation
- Friday 0800-1000, Dec 13
- Room 412, 5 Wisdom Valley

- **Discussion for Group 2**: Design Consultation
- Friday 0800-1000, Dec 13
- Room 202, 1 Lychee Park

Thank you!

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