ME303 Introduction to Mechanical Design

# Lecture 14
# Mechanical Springs

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# Agenda

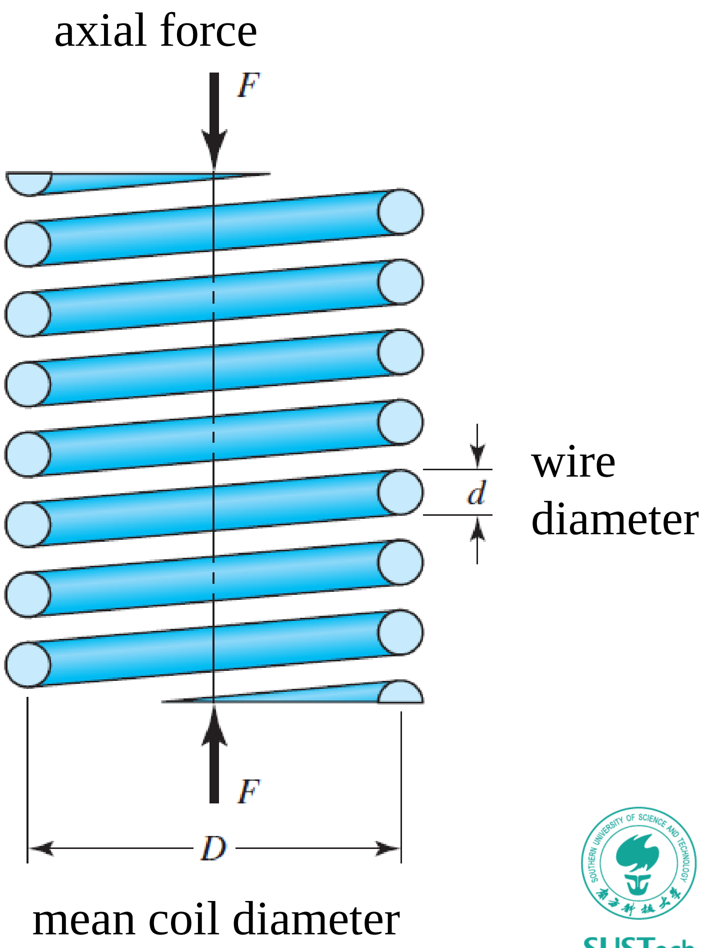
## Week 13, Wednesday, Dec 4, 2019

- Stresses in Helical Springs
- Deflection of Helical Springs
- Compression Springs
- Stability
- Spring Materials
- Helical Compression Spring Design for Static Service
- Critical Frequency of Helical Springs
- Fatigue Loading of Helical Compression Springs
- Helical Compression Spring Design for Fatigue Loading
- Extension Springs
- Helical Coil Torsion Springs
- Belleville Springs
- Miscellaneous Springs

# Mechanical Springs

- Exert force
- Provide flexibility
- Store or absorb energy

- Helical coil spring with round wire
  - Equilibrium forces at cut section anywhere in the body of the spring indicates direct shear and torsion.

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# Stresses in Helical Springs

## Torsional Shear + Direct Shear

$$\tau_{max} = \frac{Tr}{J} + \frac{F}{A}$$

$$\tau_{max} = \tau,\ T = FD/2,\ r = d/2,$$

$$J = \pi d^4/32 \qquad A = \pi d^2/4$$

$$\tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2} = \left(1 + \frac{d}{2D}\right)\left(\frac{8FD}{\pi d^3}\right)$$

spring index (4~12)

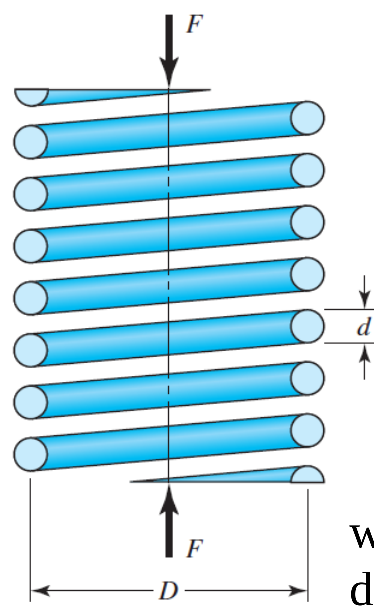
$$C = \frac{D}{d}$$

shear stress-correction factor

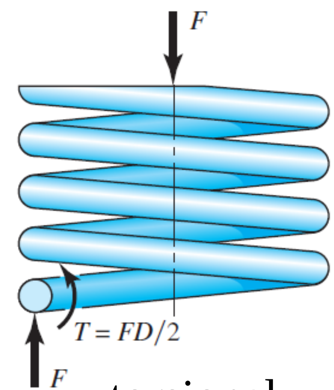
$$\tau = K_s \frac{8FD}{\pi d^3} \qquad K_s = \frac{2C+1}{2C}$$

The use of square or rectangular wire is not recommended for springs unless space limitations make it necessary

axial force




isolated section




torsional moment

wire diameter

mean coil diameter

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# Curvature Effect

Stress concentration type of effect on inner fiber due to curvature

- Can be ignored for static, ductile conditions due to localized cold-working
- Can account for effect by replacing $K_s$ with *Wahl factor* or *Bergsträsser factor (order of 1% difference)* which account for both direct shear and curvature effect

$$K_W = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$K_B = \frac{4C+2}{4C-3}$$

$$\tau = K_B \frac{8FD}{\pi d^3}$$

$$K_s = \frac{2C+1}{2C}$$

Cancelling the curvature effect to isolate the curvature factor

$$K_c = \frac{K_B}{K_s} = \frac{2C(4C+2)}{(4C-3)(2C+1)}$$



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# Deflection of Helical Springs

Total Strain Energy for a Helical Spring = Torsional + Shear

$$U = \frac{T^2 l}{2GJ} + \frac{F^2 l}{2AG}$$

Use Castigliano's method to relate force and deflection

$T = FD/2$, $l = \pi DN$, $J = \pi d^4/32$, and $A = \pi d^2/4$

$$U = \frac{4F^2 D^3 N}{d^4 G} + \frac{2F^2 DN}{d^2 G}$$

$N = N_a =$ number of active coils

$$y = \frac{\partial U}{\partial F} = \frac{8FD^3 N}{d^4 G} + \frac{4FDN}{d^2 G}$$

Direct computation (estimation) of spring rate using geometric parameters

$C = D/d$,

$$y = \frac{8FD^3 N}{d^4 G}\left(1 + \frac{1}{2C^2}\right) \approx \frac{8FD^3 N}{d^4 G}$$

spring rate

$k = F/y$,

$$k \approx \frac{d^4 G}{8D^3 N}$$

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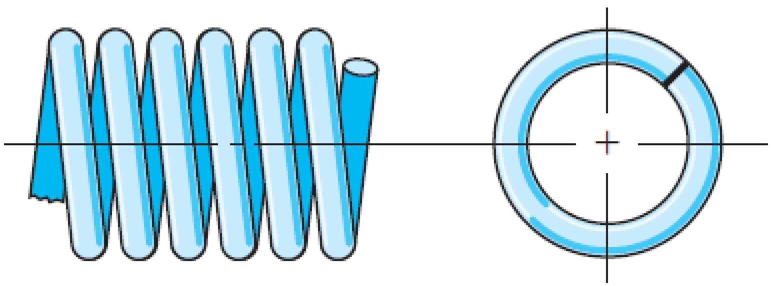
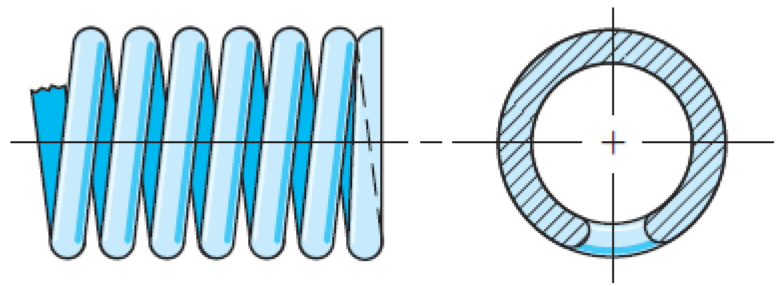
# Compression Springs

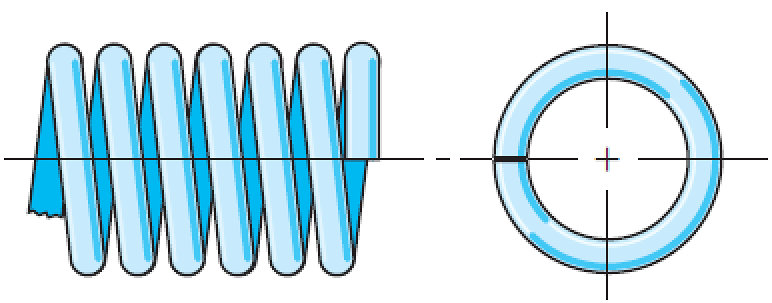
## Four Types of Common Spring End



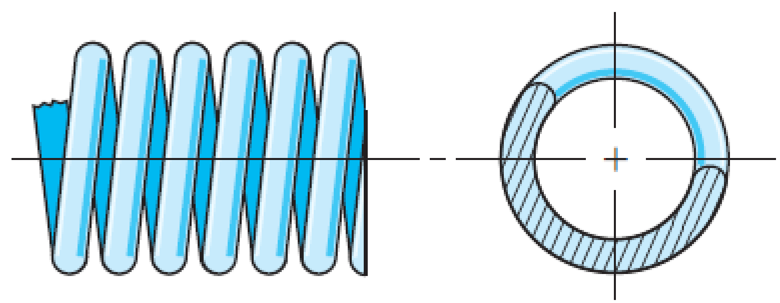
(*a*) Plain end, right hand



(*c*) Squared and ground end, left hand



(*b*) Squared or closed end, right hand



(*d*) Plain end, ground, left hand

# Compression Springs With Different Ends

## Formulas

| Term | Plain | Plain and Ground | Squared or Closed | Squared and Ground |
|---|---|---|---|---|
| | Type of Spring Ends | | | |
| End coils, $N_e$ | 0 | 1 | 2 | 2 |
| Total coils, $N_t$ | $N_a$ | $N_a + 1$ | $N_a + 2$ | $N_a + 2$ |
| Free length, $L_0$ | $pN_a + d$ | $p(N_a + 1)$ | $pN_a + 3d$ | $pN_a + 2d$ |
| Solid length, $L_s$ | $d(N_t + 1)$ | $dN_t$ | $d(N_t + 1)$ | $dN_t$ |
| Pitch, $p$ | $(L_0 - d)/N_a$ | $L_0/(N_a + 1)$ | $(L_0 - 3d)/N_a$ | $(L_0 - 2d)/N_a$ |

$N_a$ is the number of active coils

# Set Removal or Presetting

A process used in manufacturing a spring to induce useful residual stresses

- The spring is made longer than needed, then compressed to solid height, intentionally exceeding the yield strength.
- This operation sets the spring to the required final free length.
- Yielding induces residual stresses opposite in direction to those induced in service.
- 10 to 30 percent of the initial free length should be removed.
- Set removal is not recommended when springs are subject to fatigue.

# Critical Deflection for Stability

When the deflection exceeds the critical deflection

- Buckling type of instability can occur in compression springs when the deflection exceeds $y_{cr}$

$$C_1' = \frac{E}{2(E-G)}$$

$$C_2' = \frac{2\pi^2(E-G)}{2G+E}$$

*end-condition constant*

$$y_{cr} = L_0 C_1' \left[ 1 - \left( 1 - \frac{C_2'}{\lambda_{eff}^2} \right)^{1/2} \right]$$

*effective slenderness ratio*

*elastic constants*

$$\lambda_{eff} = \frac{\alpha L_0}{D}$$

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# End-Condition Constant

It accounts for the way in which the ends of the spring are supported.

| End Condition | Constant $\alpha$ |
|---|---|
| Spring supported between flat parallel surfaces (fixed ends) | 0.5 |
| One end supported by flat surface perpendicular to spring axis (fixed); other end pivoted (hinged) | 0.707 |
| Both ends pivoted (hinged) | 1 |
| One end clamped; other end free | 2 |

*Ends supported by flat surfaces must be squared and ground.

*effective slenderness ratio*

*elastic constants* → $\alpha$

$$\lambda_{\text{eff}} = \frac{\alpha L_0}{D}$$

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# Absolute Stability

- Absolute stability occurs when

$$y_{cr} = L_0 C_1' \left[ 1 - \left( 1 - \frac{C_2'}{\lambda_{\text{eff}}^2} \right)^{1/2} \right]$$

$$C_2' / \lambda_{\text{eff}}^2 > 1$$

- This results in the condition for absolute stability

$$L_0 < \frac{\pi D}{\alpha} \left[ \frac{2(E - G)}{2G + E} \right]^{1/2}$$

- For steels, this turns out to be

$$L_0 < 2.63 \frac{D}{\alpha}$$

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# Spring Materials

## Some Common Spring Steels

- Hard-drawn wire (0.60-0.70C)
  - Cheapest general-purpose
  - Use only where life, accuracy, and deflection are not too important
- Oil-tempered wire (0.60-0.70C)
  - General-purpose
  - Heat treated for greater strength and uniformity of properties
  - Often used for larger diameter spring wire
- Music wire (0.80-0.95C)
  - Higher carbon for higher strength
  - Best, toughest, and most widely used for small springs
  - Good for fatigue
- Chrome-vanadium
  - Popular alloy spring steel
  - Higher strengths than plain carbon steels
  - Good for fatigue, shock, and impact
- Chrome-silicon
  - Good for high stresses, long fatigue life, and shock

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# Strength of Spring Materials

With small wire diameters, strength is a function of diameter.

- Tensile strength vs. wire diameter is almost a straight line on log-log scale.
- The equation of this line is $S_{ut} = \dfrac{A}{d^m}$, where $A$ is the intercept and $m$ is the slope.
- For common spring steels

| Material | ASTM No. | Exponent $m$ | Diameter, in | $A$, kpsi · in$^m$ | Diameter, mm | $A$, MPa · mm$^m$ | Relative Cost of Wire |
|---|---|---|---|---|---|---|---|
| Music wire* | A228 | 0.145 | 0.004–0.256 | 201 | 0.10–6.5 | 2211 | 2.6 |
| OQ&T wire† | A229 | 0.187 | 0.020–0.500 | 147 | 0.5–12.7 | 1855 | 1.3 |
| Hard-drawn wire‡ | A227 | 0.190 | 0.028–0.500 | 140 | 0.7–12.7 | 1783 | 1.0 |
| Chrome-vanadium wire§ | A232 | 0.168 | 0.032–0.437 | 169 | 0.8–11.1 | 2005 | 3.1 |
| Chrome-silicon wire‖ | A401 | 0.108 | 0.063–0.375 | 202 | 1.6–9.5 | 1974 | 4.0 |
| 302 Stainless wire# | A313 | 0.146 | 0.013–0.10 | 169 | 0.3–2.5 | 1867 | 7.6–11 |
| | | 0.263 | 0.10–0.20 | 128 | 2.5–5 | 2065 | |
| | | 0.478 | 0.20–0.40 | 90 | 5–10 | 2911 | |
| Phosphor-bronze wire** | B159 | 0 | 0.004–0.022 | 145 | 0.1–0.6 | 1000 | 8.0 |
| | | 0.028 | 0.022–0.075 | 121 | 0.6–2 | 913 | |
| | | 0.064 | 0.075–0.30 | 110 | 2–7.5 | 932 | |

# Estimating Torsional Yield Strength

Since helical springs experience shear stress, shear yield strength is needed.

- If actual data is not available, estimate from tensile strength
- Assume yield strength is between 60-90% of tensile strength

$$0.6S_{ut} \le S_{sy} \le 0.9S_{ut}$$

- Assume the distortion energy theory can be employed to relate the shear strength to the normal strength.

$$S_{sy} = 0.577S_y$$

- This results in $0.35S_{ut} \le S_{sy} \le 0.52S_{ut}$

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# Mechanical Properties of Some Spring Wires

| Material | Elastic Limit, Percent of $S_{ut}$ Tension | Elastic Limit, Percent of $S_{ut}$ Torsion | Diameter $d$, in | $E$ Mpsi | $E$ GPa | $G$ Mpsi | $G$ GPa |
|---|---|---|---|---|---|---|---|
| Music wire A228 | 65–75 | 45–60 | <0.032 | 29.5 | 203.4 | 12.0 | 82.7 |
| | | | 0.033–0.063 | 29.0 | 200 | 11.85 | 81.7 |
| | | | 0.064–0.125 | 28.5 | 196.5 | 11.75 | 81.0 |
| | | | >0.125 | 28.0 | 193 | 11.6 | 80.0 |
| HD spring A227 | 60–70 | 45–55 | <0.032 | 28.8 | 198.6 | 11.7 | 80.7 |
| | | | 0.033–0.063 | 28.7 | 197.9 | 11.6 | 80.0 |
| | | | 0.064–0.125 | 28.6 | 197.2 | 11.5 | 79.3 |
| | | | >0.125 | 28.5 | 196.5 | 11.4 | 78.6 |
| Oil tempered A239 | 85–90 | 45–50 | | 28.5 | 196.5 | 11.2 | 77.2 |
| Valve spring A230 | 85–90 | 50–60 | | 29.5 | 203.4 | 11.2 | 77.2 |
| Chrome-vanadium A231 | 88–93 | 65–75 | | 29.5 | 203.4 | 11.2 | 77.2 |
| A232 | 88–93 | | | 29.5 | 203.4 | 11.2 | 77.2 |
| Chrome-silicon A401 | 85–93 | 65–75 | | 29.5 | 203.4 | 11.2 | 77.2 |
| Stainless steel | | | | | | | |
| A313* | 65–75 | 45–55 | | 28 | 193 | 10 | 69.0 |
| 17-7PH | 75–80 | 55–60 | | 29.5 | 208.4 | 11 | 75.8 |
| 414 | 65–70 | 42–55 | | 29 | 200 | 11.2 | 77.2 |
| 420 | 65–75 | 45–55 | | 29 | 200 | 11.2 | 77.2 |

# Maximum Allowable Torsional Stresses

## For Helical Compression Springs in Static Applications

| Material | Maximum Percent of Tensile Strength | |
|---|---|---|
| | Before Set Removed (includes $K_W$ or $K_B$) | After Set Removed (includes $K_s$) |
| Music wire and cold-drawn carbon steel | 45 | 60–70 |
| Hardened and tempered carbon and low-alloy steel | 50 | 65–75 |
| Austenitic stainless steels | 35 | 55–65 |
| Nonferrous alloys | 35 | 55–65 |

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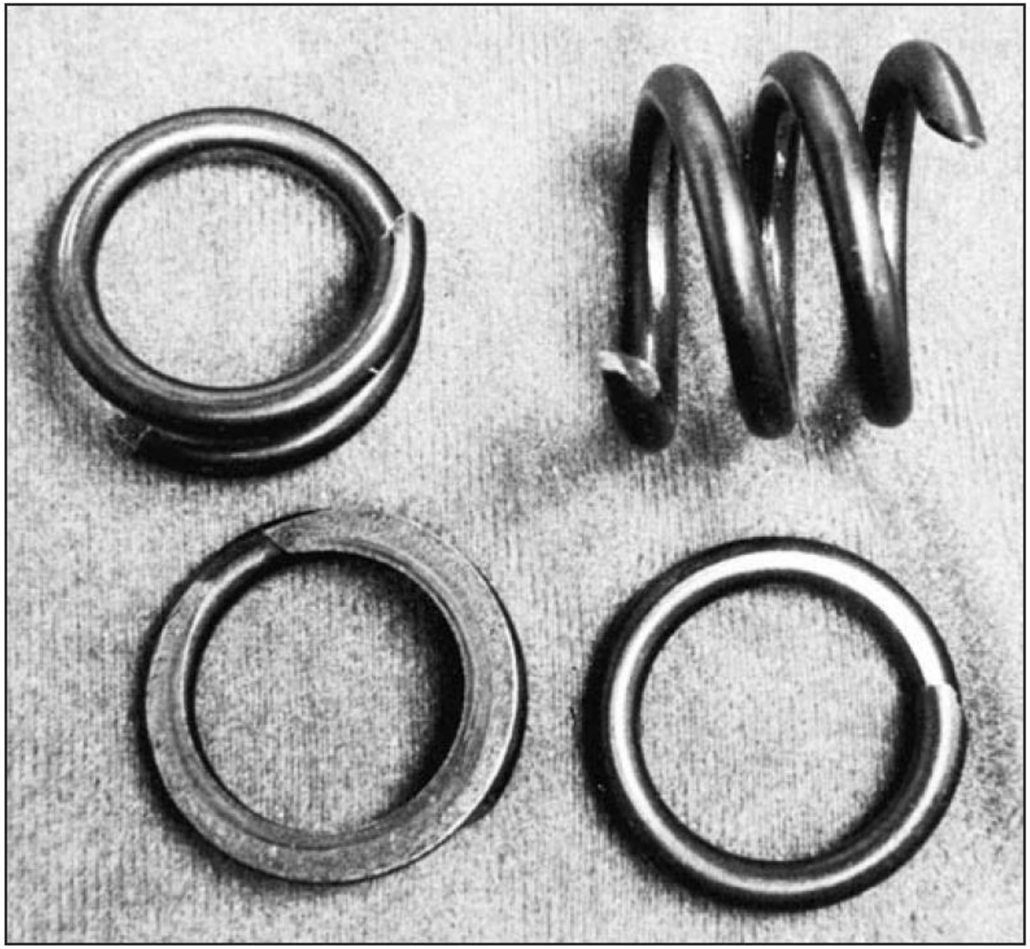
# Critical Frequency of Helical Springs

When one end of a spring is displaced rapidly, a wave called a spring surge travels down the spring.

- If the other end is fixed, the wave can reflect back.
- If the wave frequency is near the natural frequency of the spring, resonance may occur resulting in extremely high stresses.
- Catastrophic failure may occur, as shown in this valve-spring from an over-revved engine.



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# Critical Frequency of Helical Springs

When one end of a spring is displaced rapidly, a wave called a spring surge travels down the spring.

- The governing equation is the wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{W}{kgl^2}\frac{\partial^2 u}{\partial t^2}$$

where $k$ = spring rate

$g$ = acceleration due to gravity

$l$ = length of spring

$W$ = weight of spring

$x$ = coordinate along length of spring

$u$ = motion of any particle at distance $x$

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# Critical Frequency of Helical Springs

When one end of a spring is displaced rapidly, a wave called a spring surge travels down the spring.

- The solution to this equation is harmonic and depends on the given physical properties as well as the end conditions.

$$\omega = m\pi\sqrt{\frac{kg}{W}} \qquad m = 1, 2, 3, \ldots$$

- In cycles per second, or hertz, $f = \frac{1}{2}\sqrt{\frac{kg}{W}}$
- With one end against a flat plate and the other end free, $f = \frac{1}{4}\sqrt{\frac{kg}{W}}$
- The weight of a helical spring is $W = AL\gamma = \frac{\pi d^2}{4}(\pi D N_a)(\gamma) = \frac{\pi^2 d^2 D N_a \gamma}{4}$
- The fundamental critical frequency should be greater than 15 to 20 times the frequency of the force or motion of the spring.
- If necessary, redesign the spring to increase *k* or decrease *W*.

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# Fatigue Loading of Helical Compression Springs

Size, material, and tensile strength have no effect on the endurance limits of spring steels in sizes under 10 mm.

- Testing found the endurance strength components for infinite life to be

Unpeened:

$$S_{sa} = 35 \text{ kpsi } (241 \text{ MPa}) \qquad S_{sm} = 55 \text{ kpsi } (379 \text{ MPa})$$

Peened:

$$S_{sa} = 57.5 \text{ kpsi } (398 \text{ MPa}) \qquad S_{sm} = 77.5 \text{ kpsi } (534 \text{ MPa})$$

- These constant values are used with Gerber or Goodman failure criteria to find the endurance limit.

# Fatigue Loading of Helical Compression Springs

Size, material, and tensile strength have no effect on the endurance limits of spring steels in sizes under 10 mm.

- For example, with an unpeened spring with $S_{su}$ = 211.5 kpsi, the Gerber ordinate intercept for shear, from Eq. (6-42), is

$$S_{se} = \frac{S_{sa}}{1 - \left(\frac{S_{sm}}{S_{su}}\right)^2} = \frac{35}{1 - \left(\frac{55}{211.5}\right)^2} = 37.5 \text{ kpsi}$$

- For the Goodman criterion, it would be $S_{se}$ = 47.3 kpsi.
- Each possible wire size would change the endurance limit since $S_{su}$ is a function of wire size.
- It has been found that for polished, notch-free, cylindrical specimens subjected to torsional shear stress, the maximum alternating stress that may be imposed is constant and independent of the mean stress.
  - Many compression springs approach these conditions.
  - This failure criterion is known as the *Sines failure criterion*.

# Helical Compression Spring

## Design for Fatigue Loading

- From the standard approach, the alternating and midrange forces are

$$F_a = \frac{F_{\max} - F_{\min}}{2}$$

$$F_m = \frac{F_{\max} + F_{\min}}{2}$$

- The alternating and midrange stresses are

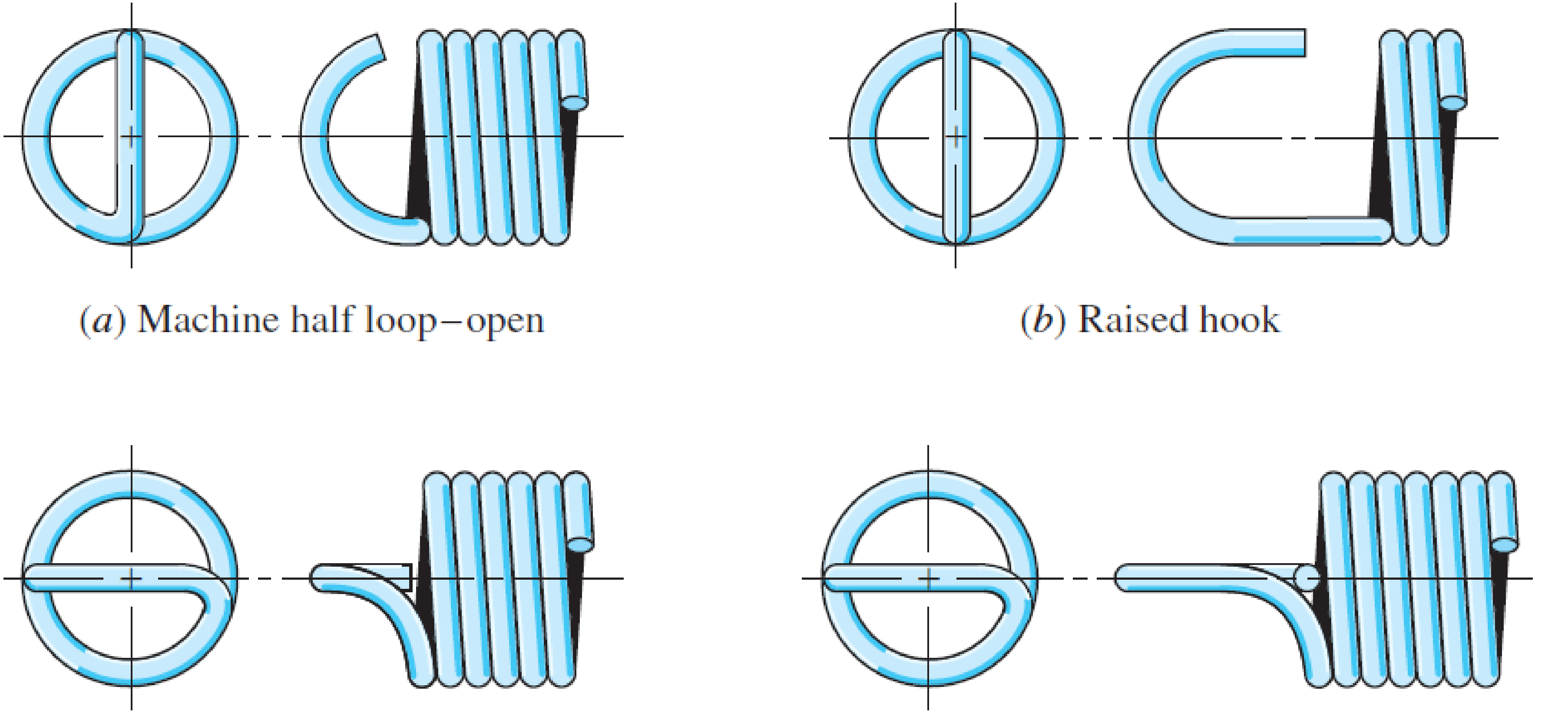
$$\tau_a = K_B \frac{8 F_a D}{\pi d^3}$$

$$\tau_m = K_B \frac{8 F_m D}{\pi d^3}$$

# Extension Springs

Extension springs are similar to compression springs within the body of the spring.

- Hooks are needed at the ends of the springs to apply tensile loads



(*a*) Machine half loop–open (*b*) Raised hook

(*c*) Short twisted loop (*d*) Full twisted loop

# Stress in the Hook

- In a typical hook, a critical stress location is at point A, where there is bending and axial loading.

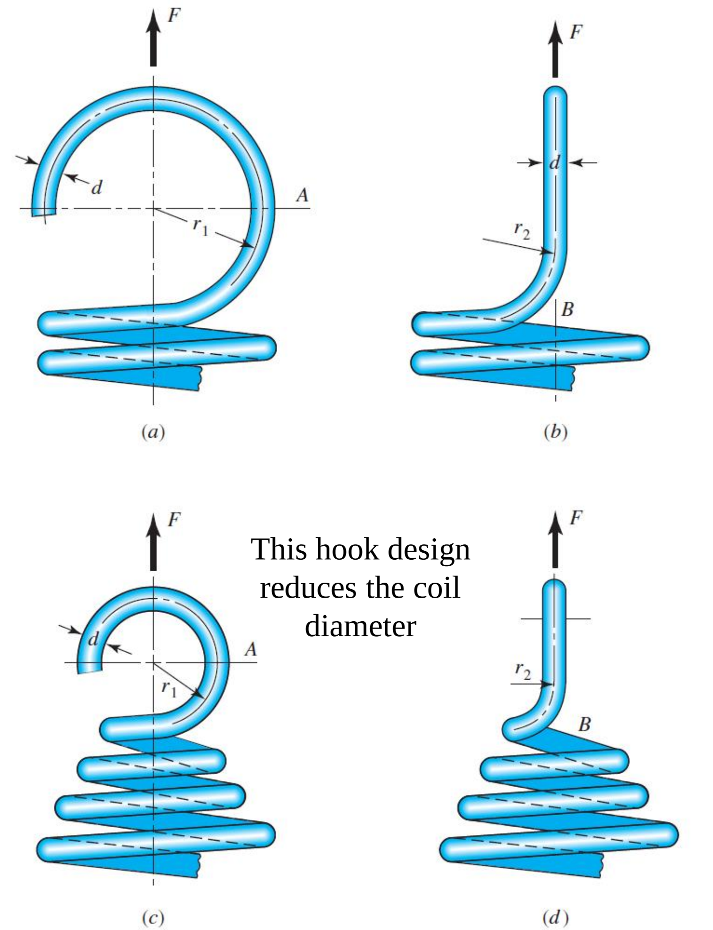
$$\sigma_A = F\left[(K)_A \frac{16D}{\pi d^3} + \frac{4}{\pi d^2}\right]$$

a bending stress-correction factor for curvature

$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} \qquad C_1 = \frac{2r_1}{d}$$

- Another potentially critical stress location is at point B, where there is primarily torsion.

$$\tau_B = (K)_B \frac{8FD}{\pi d^3} \qquad (K)_B = \frac{4C_2 - 1}{4C_2 - 4} \qquad C_2 = \frac{2r_2}{d}$$



# Close-wound Extension Springs

Extension springs are often made with coils in contact with one another, called close-wound.

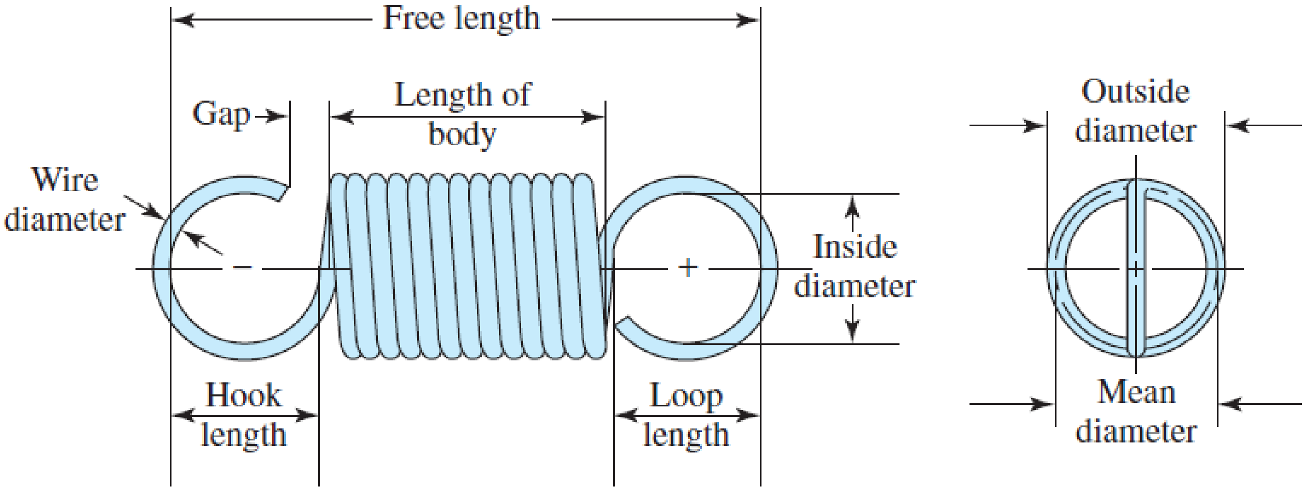
- Including some initial tension in close-wound springs helps hold the free length more accurately. $F = F_i + ky$
  - The load-deflection curve is offset by this initial tension $F_i$
- The free length is measured inside the end hooks

$$L_0 = 2(D - d) + (N_b + 1)d = (2C - 1 + N_b)d$$

The hooks contribute to the spring rate

$$N_a = N_b + \frac{G}{E}$$

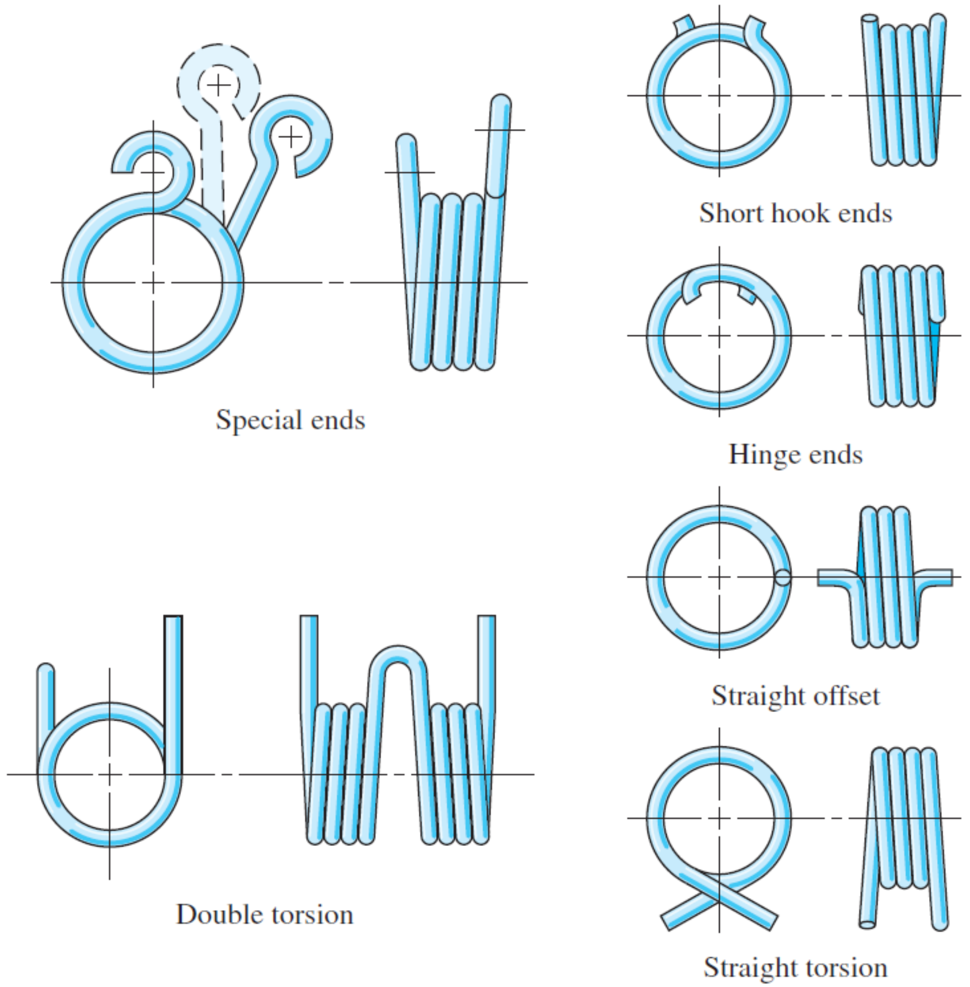
an equivalent number of active coils



# Helical Coil Torsion Spring

When a helical coil spring is subjected to end torsion, it is called a *torsion spring*.

- The wire in a torsion spring is in bending.
- The springs are designed to wind tighter in service.
    - As the applied torque increases, the inside diameter of the coil decreases.
- Care must be taken so that the coils do not interfere with the pin, rod, or arbor.
- The bending mode in the coil might seem to invite square- or rectangular-cross section wire,
    - but cost, range of materials, and availability discourage its use.

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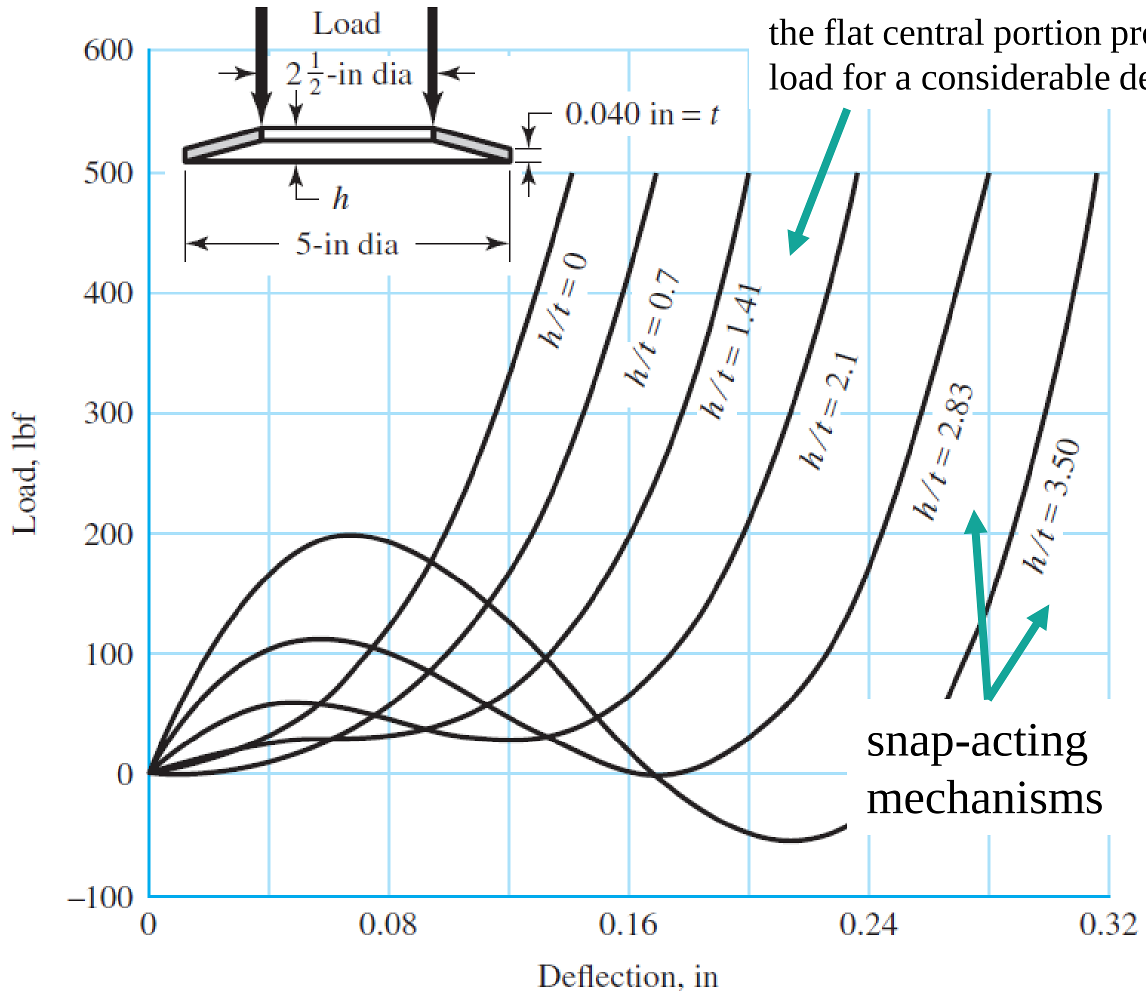
# Belleville Springs

## A coned-disk spring with a non-linear spring constant




- Occupies only a small space.
- Variation in the *h/t* ratio will produce a wide variety of load-deflection curve shapes.

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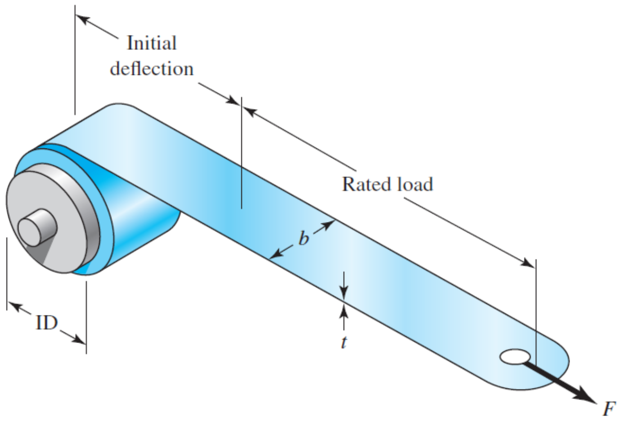
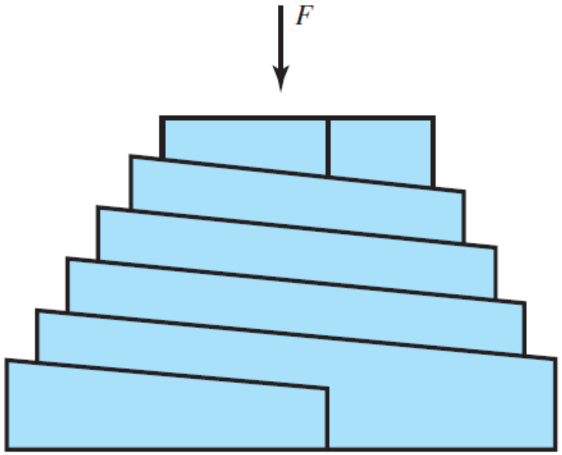
# Miscellaneous Springs

## Constant-Force Springs | Volute Spring

- The extension spring shown is made of slightly curved strip steel, not flat. The fore required to uncoil it remains constant, thus *Constant-Force Spring*.
- A *Volute Spring* is a conical spring made from a wide, thin strip, or "flat", of material wound on the flat so that the coils fit inside one another.

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Project 3: Shaft Design

- Online at course website

Next class

- **Lab for Group 2**: Design Consultation
- Friday 0800-1000, Dec 6
- Room 412, 5 Wisdom Valley

- **Discussion for Group 1**: Design Consultation
- Friday 0800-1000, Dec 6
- Room 202, 1 Lychee Park

# Thank you!


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